Learning about Risk and Return: A Simple Model of Bubbles and Crashes

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Role of Risk in Financial Bubbles:

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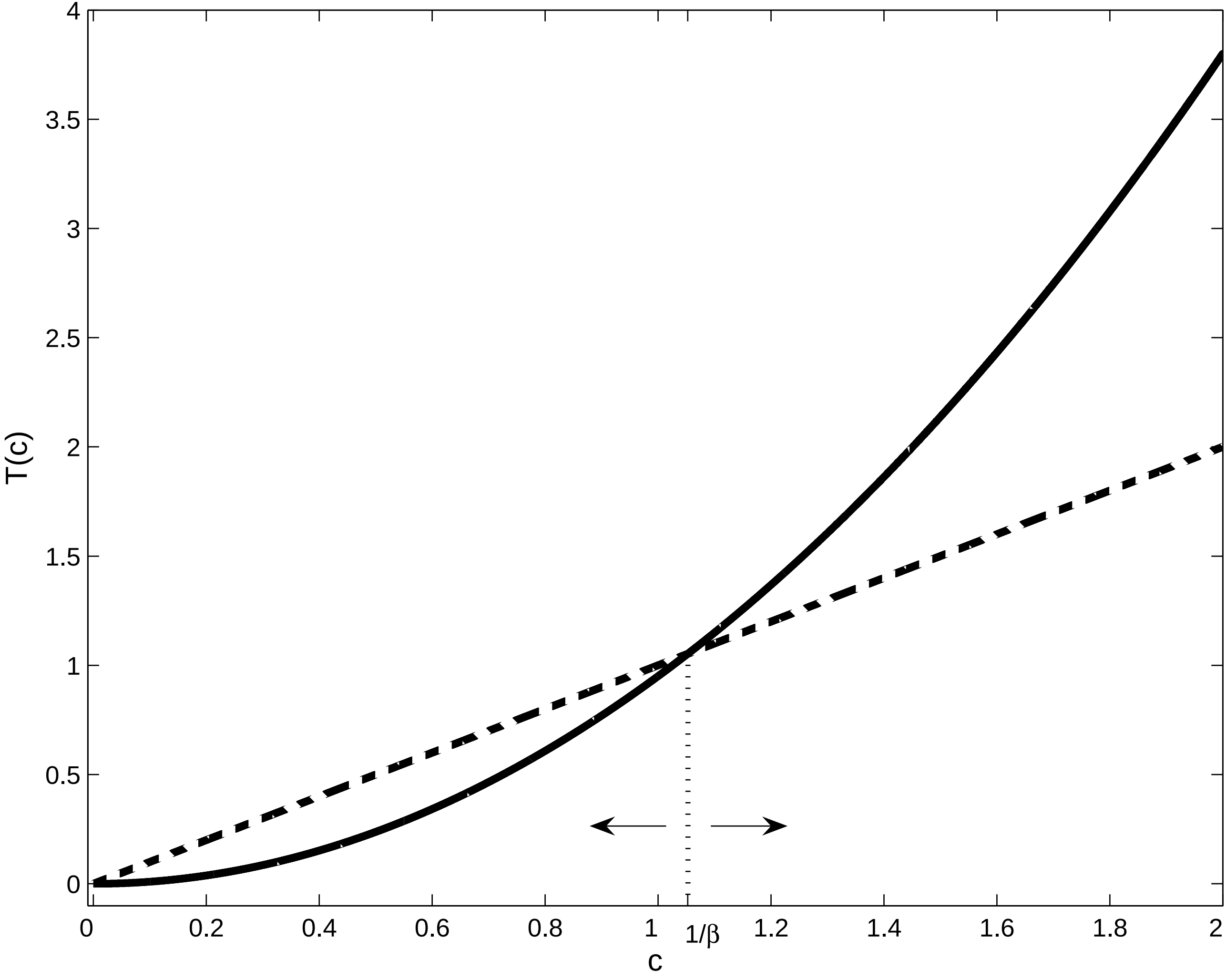
Risk-neutral asset-pricing model:

$$p_t = \alpha + \beta E_t p_{t+1} + \beta d_t$$

has rational expectations solutions of form:

$$p_t = a + b p_{t-1} + c d_t + f \xi_t$$

Fundamentals: $b = 0, f = 0$ Bubbles: $b = \beta^{-1} > 1 \Rightarrow$ explosive.



T(c)
c
1/β
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0.5
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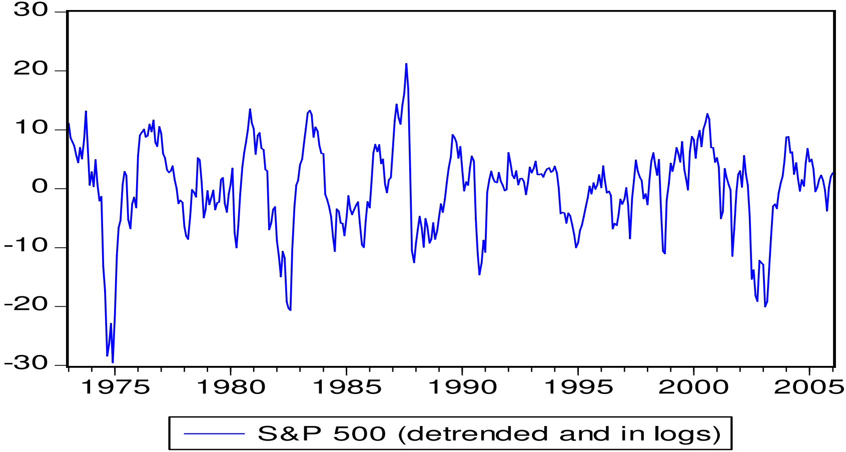
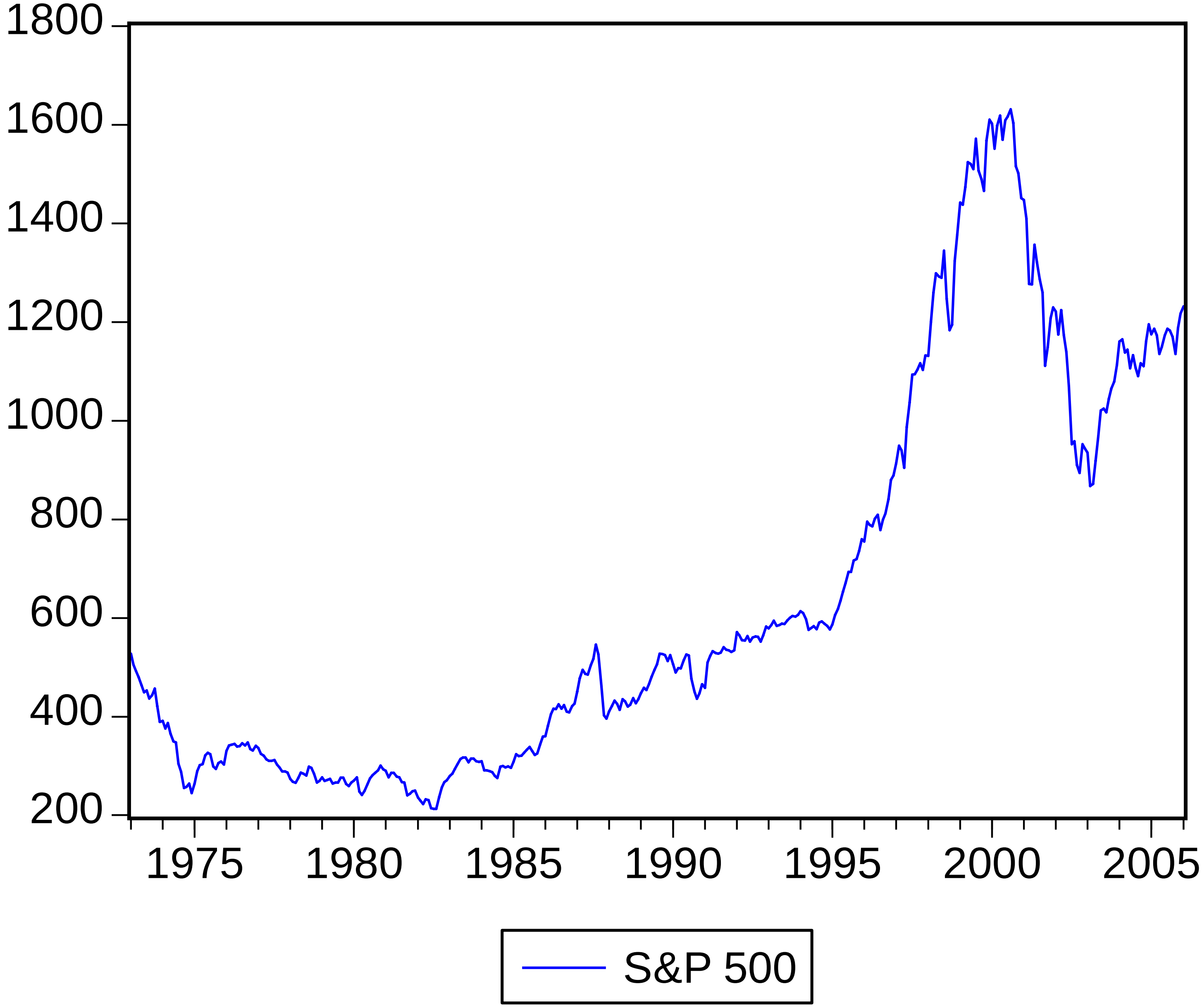
2

But, bubbles seem to exist...(?)

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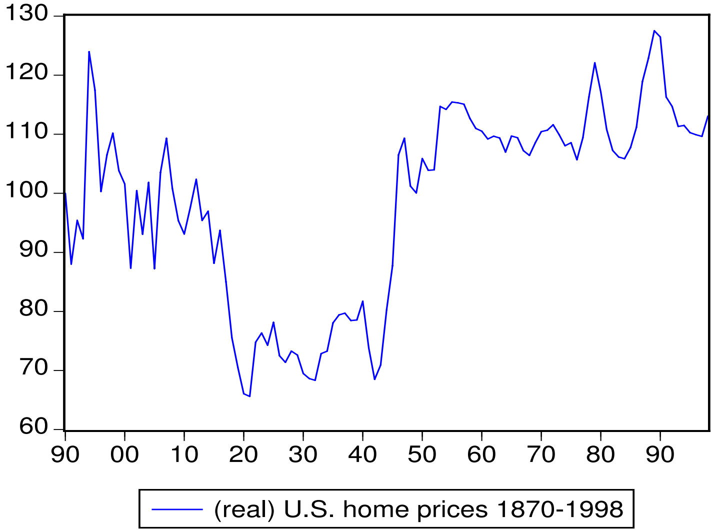
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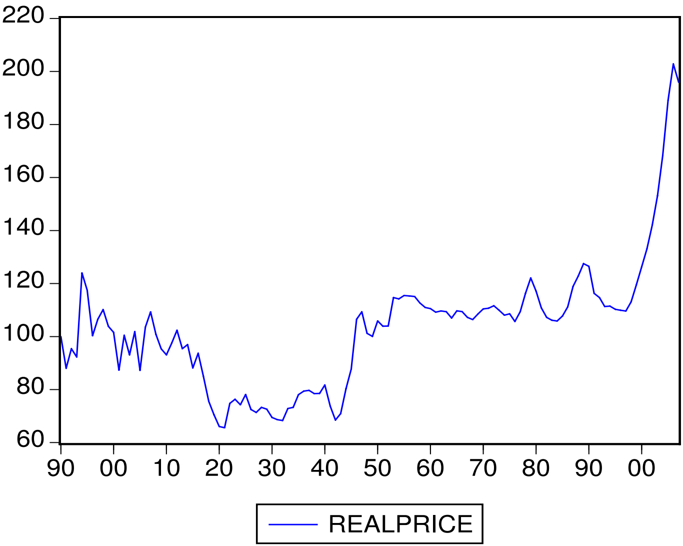
(real) U.S. home prices 1870-1998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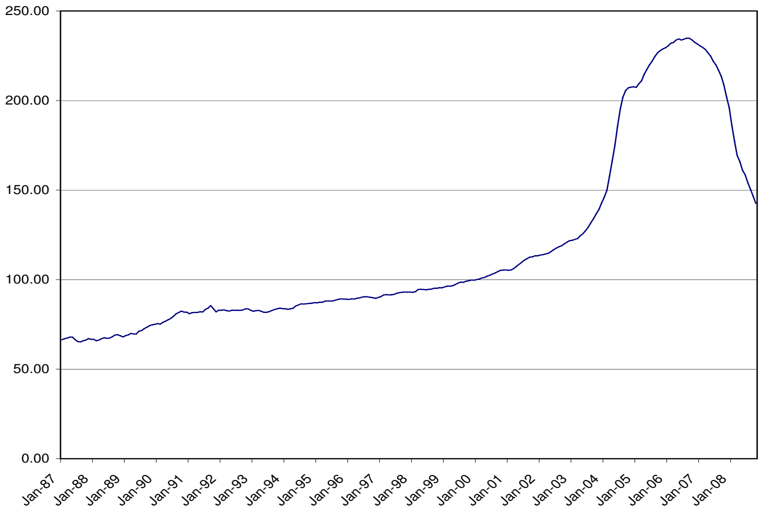
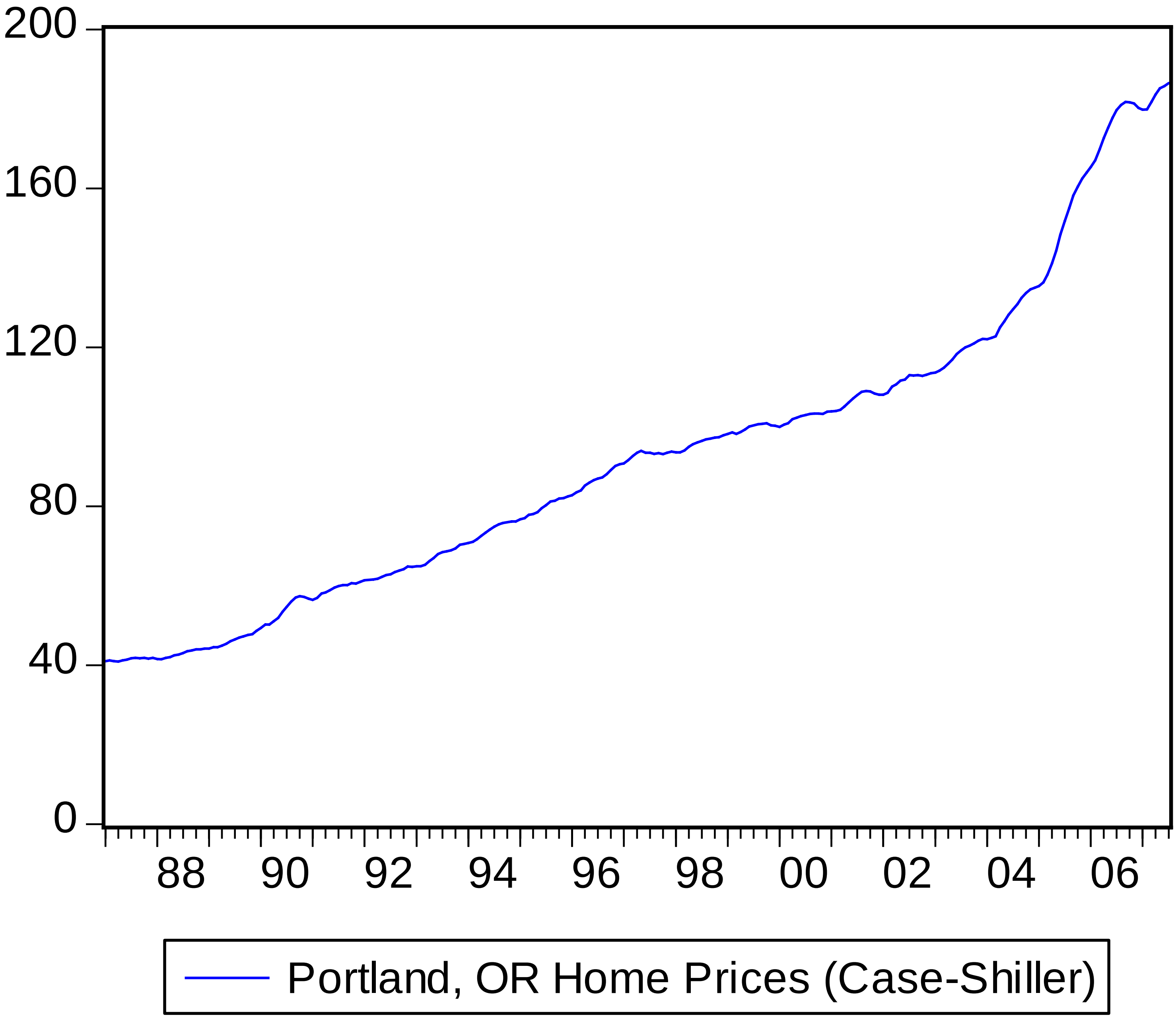
REALPRICE

Las Vegas Home Prices

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02
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06
Portland, OR Home Prices (Case-Shiller)

This paper:

- Takes a simple asset pricing model with linear demand for the risky asset and risk-aversion.
- Risk-aversion implies asset price depends on expected conditional variance of net returns. But, bubbles REE is still unstable under least squares learning.
- Constant gain learning version of the model:
  - creates drift that causes agents' to occasionally believe stock prices follow a random walk: perceive shocks to be permanent;
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- Our story: risk, combined with adaptive learning, leads to recurring bubbles and crashes.

Model

- Risky asset yields dividends $\{y_t\}$, trades at price p_t.
- Risk free asset pays $R = \beta^{-1} > 1$.
- Mean-variance setting yields demand for risky asset

$$z_{dt} = \frac{E_t^* \left(p_{t+1} + y_{t+1}\right) - \beta^{-1} p_t}{a\sigma_t^2}$$

where σ_t^2 is agents' cond'l expect. of $p_{t+1} + y_{t+1}$.

Model (cont.)

In equilibrium, $z_{st} = z_{dt}$, implies

$$p_t = \beta \hat{E}_t (p_{t+1} + y_{t+1}) - \beta a \sigma^2 z_{st} \tag{1}$$

Assume

$$\begin{aligned} y_t &= y_0 + u_t \\ z_{st} &= \{\min(s_0, \Phi p_t)\} \cdot (1 + v_t) \\ \sigma^2 &= \hat{Var}_t (p_{t+1} + y_{t+1} - Rp_t) \end{aligned}$$

where v_t, u_t uncorrelated white noise shocks, $\Phi = 10 s_0 / \bar{p}$.

z_{st} is share supply which is:

- exogenous (constant plus white noise);
- or, endogenous when price falls to 10% of its fundamental value

Definition. A *Rational Expectations Equilibrium* is a solution to (1).

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Stability under learning

- To study E-stability, posit a perceived law of motion (PLM)

$$p_t = k + cp_{t-1} + \varepsilon_t$$

Forming $E_t p_{t+1} = k(1 + c) + c^2 p_{t-1}$ leads to the actual law of motion (ALM)

$$p_t = T(k, c)'(1, p_{t-1})' - \beta a \sigma^2 v_t, \quad s_0 \leq \Phi p_t$$

where $T(k, c) = (\beta[y_0 + k(1 + c) - a\sigma^2 s_0], \beta c^2)'$.

- But, σ^2 is an equilibrium object. Thus, also need to pin down

$$\sigma^2 = E_t\left(p_{t+1} - E_t p_{t+1} + y_{t+1} - E_t y_{t+1}\right)^2$$

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Stability of REE

Define $\theta_t = (k_t, c_t)', X_t = (1, p_{t-1})$. We assume learning takes place via the recursive algorithm

$$\begin{aligned}
\theta_t &= \theta_{t-1} + \gamma_{1,t} R_t^{-1} X_t \left(p_t - \theta_{t-1}' X_t\right) \\
R_t &= R_{t-1} + \gamma_{1,t} \left(X_t X_t' - R_{t-1}\right) \\
\sigma_t^2 &= \sigma_{t-1}^2 + \gamma_{2,t} \left(\left(p_t - \theta_{t-1}' X_t + u_t\right)^2 - \sigma_{t-1}^2\right)
\end{aligned}$$

$\gamma_{1,t}, \gamma_{2,t}$ are gain sequences, usually $\gamma_t > 0, \sum_{t \geq 0} \gamma_t = \infty$.

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Question of interest: what is the asymptotic behavior of θ_t, σ_t?

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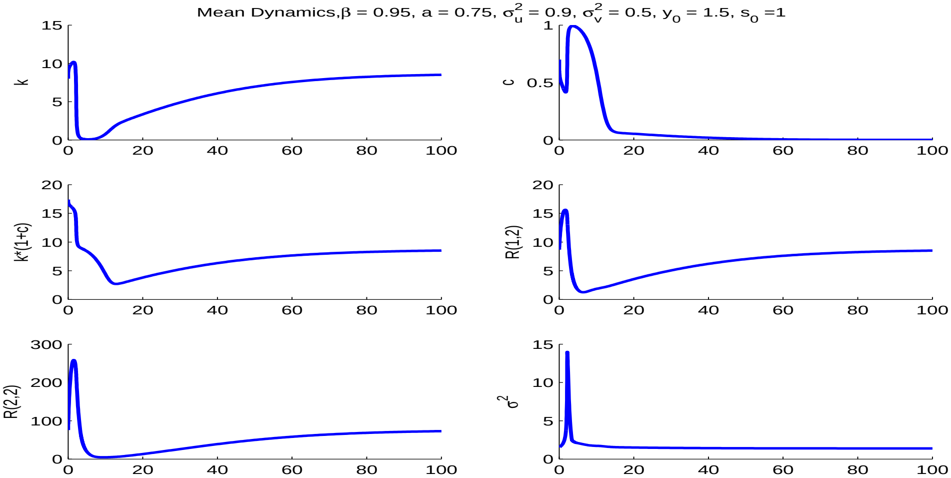
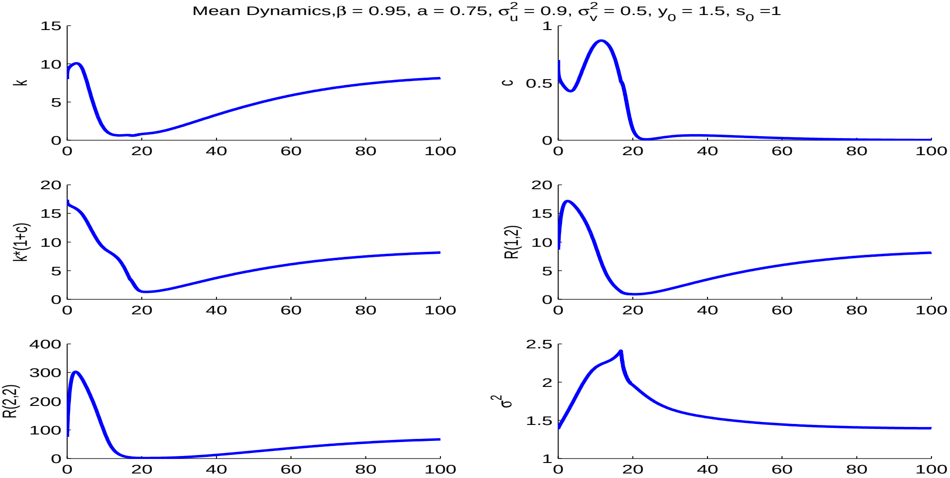
REE Stability Result

1. The fundamentals REE is stable under adaptive learning provided $0 < \beta < 1$ and $0 < \sigma^2 < 1/2\beta a\sigma_v^2$. The bubbles REE is unstable under adaptive learning.
2. As $\gamma \to 0$, constant gain learning dynamics weakly converge to the solution of the ODE.

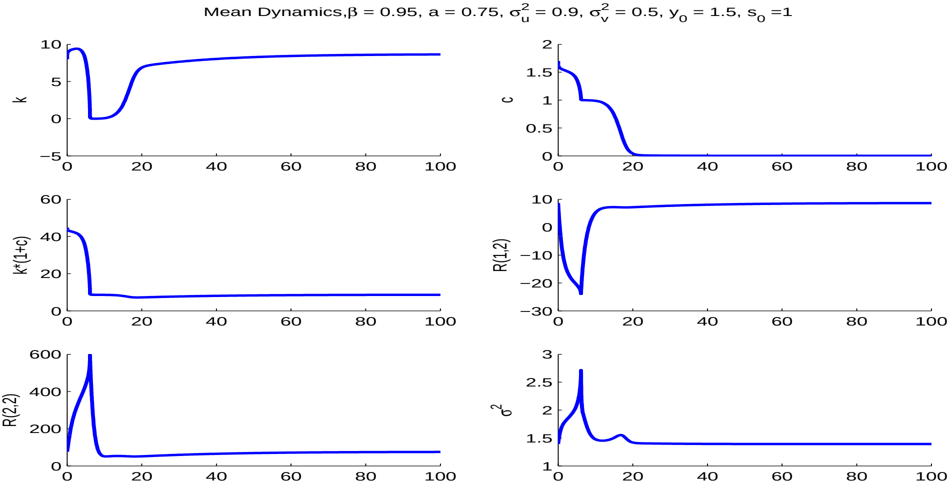
Mean Dynamics suggest occasional escapes to random walk like behavior:



Depends on relative gains:

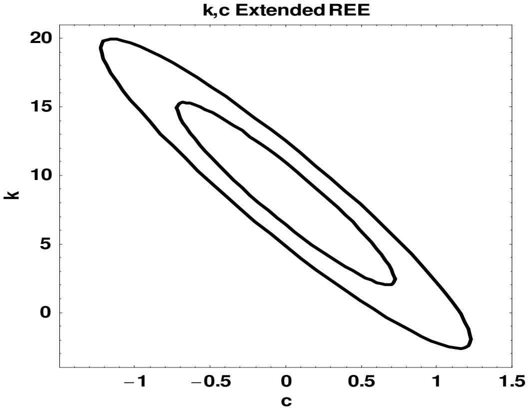


Endogenous Projection Facility:



Intuition for escapes I.

Confidence ellipses around fundamentals REE are pointed toward a random walk:



Intuition for escapes II.

- Random walk beliefs, lag coeff$\rightarrow 1$, implies PLM:

$$p_t = \frac{1}{1-L}\varepsilon_t \equiv g(L)\varepsilon_t$$

- So ALM becomes

$$p_t = \mu + f(L)\nu_t,$$

$f(L) = \beta a\sigma^2(1-\beta L)^{-1}$.

- As Sargent notes:
  1. Random walk beliefs introduce serial correlation.
  2. Random walk beliefs track constants well, so use higher-order moments to track low frequency movements. Risk induces drift in constant.
- Mean dynamics show that σ^2 triggers "escapes"

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Numerical Simulations

- Fundamentals REE is locally stable. Bubbles REE is unstable. What happens with constant gain learning? Turn to numerical simulations.
- Choose parameters
  $\beta = .95, a = .75, \sigma_v^2 = .5, \sigma_u^2 = 0.9, y_0 = 1.5, s_0 = 1.$

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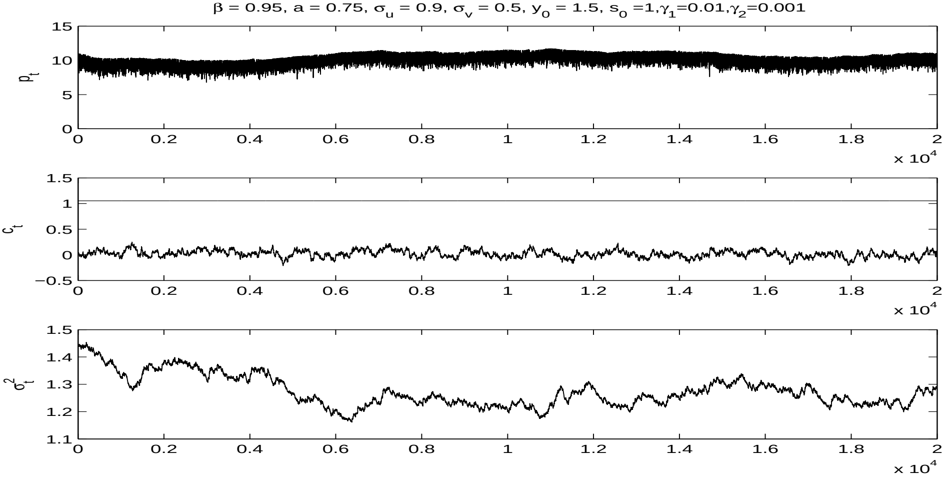
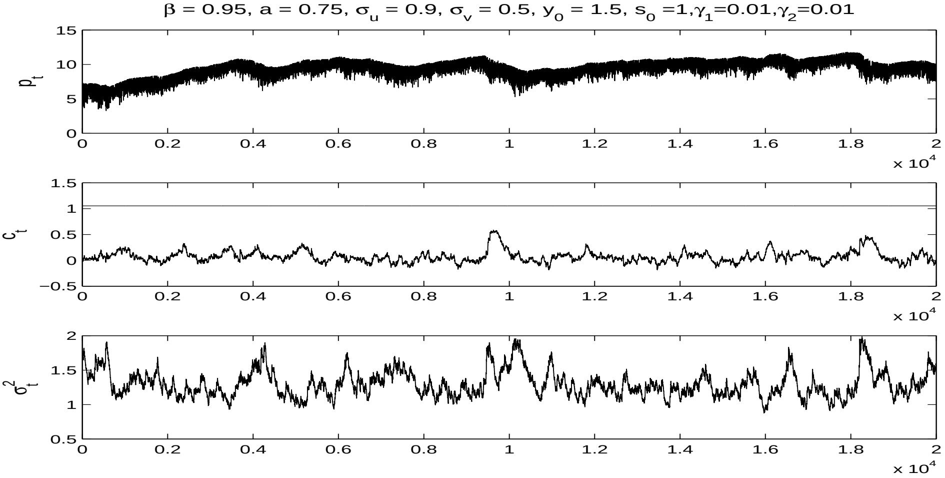
Small Gains

Figure: Constant gain learning with $\gamma_1 = .01, \gamma_2 = .001$.



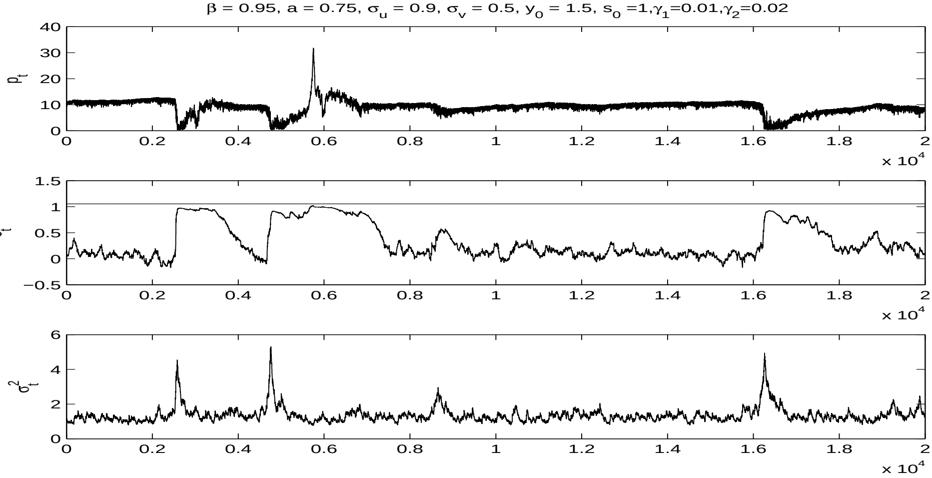
Start to see serial correlation

Figure: Constant gain learning with $\gamma_1 = \gamma_2 = .01$.



Bubbles and Crashes

Figure: Constant gain learning with $\gamma_1 = .01, \gamma_2 = .02$.



Bubbles and Crashes: Risk

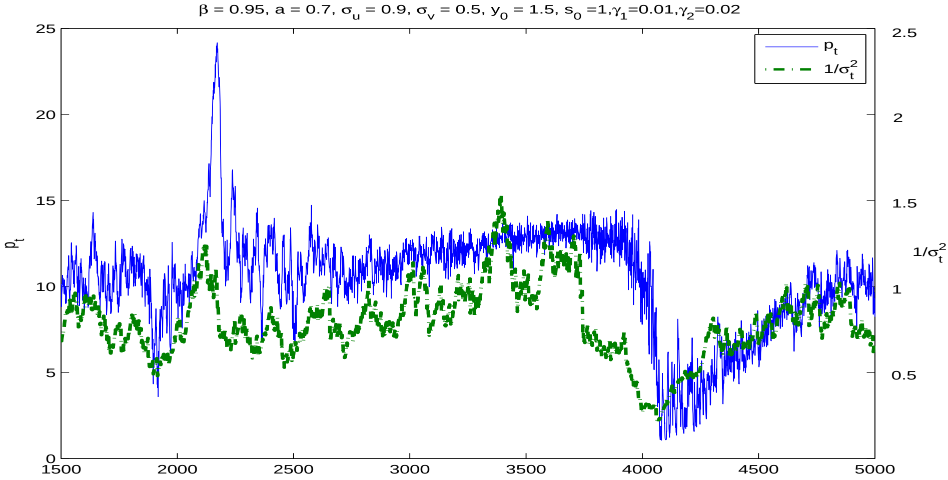
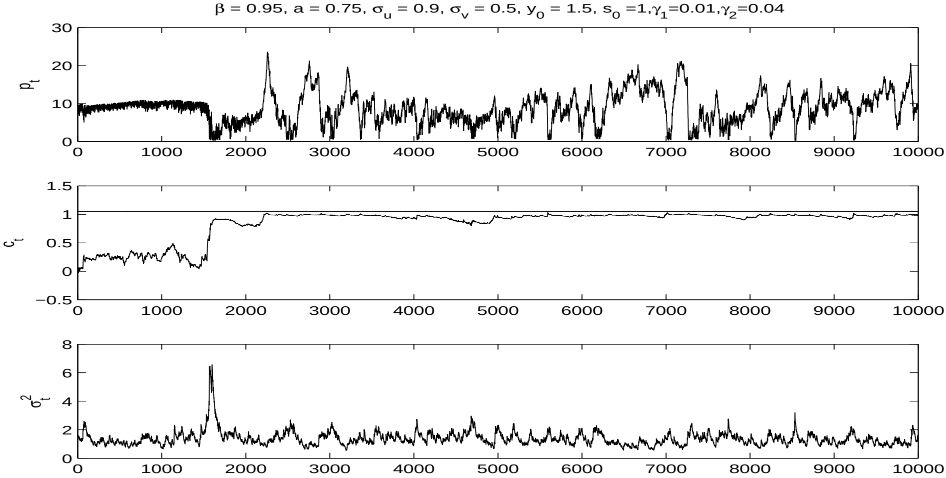


Figure: Comparing price with inverse of risk, $1/\sigma_5^2$

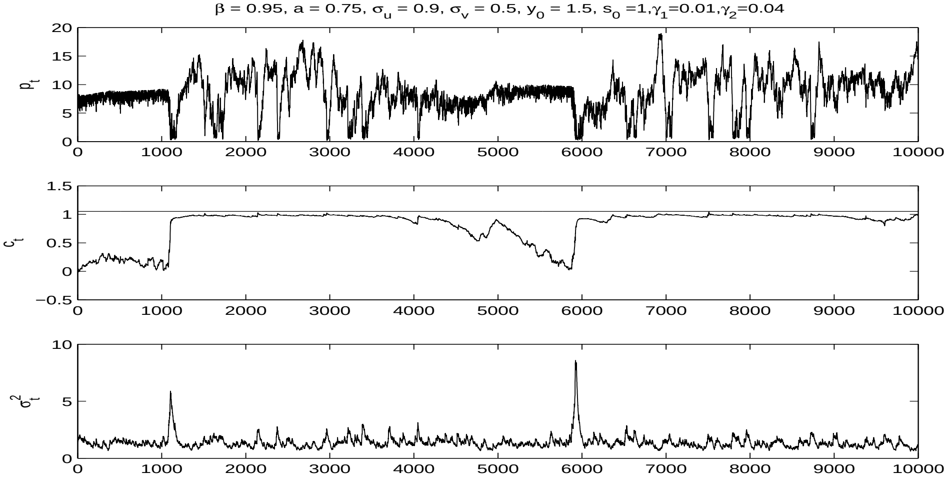
Bubbles and Crashes

Figure: Constant gain learning with $\gamma_1 = .01, \gamma_2 = .04$.



Bubbles and Crashes

Figure: Constant gain learning with $\gamma_1 = .01, \gamma_2 = .04$.



Detecting Bubbles

- Literature on testing bubbles work with regressions like

  $$p_t = a + by_t + u_t$$

  and look for unit roots in the residuals u_t. Periodically collapsing bubbles, though, will fail unit root tests. (Recursive unit root tests of Phillips, Wu, Yu (2007) an exception.)
- Recursive Estimates Statistic of Wu and Xiao: Look for exploding segments of time-series:

  $$\max_{k=1,\dots,n} \frac{k}{\sqrt{n}} \left| \frac{1}{k} \sum_{t=1}^{k} \hat{u}_t - \frac{1}{n} \sum_{t=1}^{n} \hat{u}_t \right|$$

- Simulate model for 15000 periods, storing last 1000 or 100 periods, estimate Augmented Dickey-Fuller and Recursive Statistic, repeat 1000 times.

	Simulation Length: 1000		Simulation Length: 100	
Gain Parameters	RES	ADF	RES	ADF
$\gamma_1 = .001, \gamma_2 = .001$	0.024	0.000	0.0000	0.000
$\gamma_1 = .01, \gamma_2 = .01$	0.8760	0.0040	0.0220	0.000
$\gamma_1 = .01, \gamma_2 = .04$	0.980	0.0210	0.8340	.0640

Conclusion:

- Asset pricing models should incorporate learning about risk and return.
- Suggests a story for bubbles:
  - Stock price “usually” near its fundamentals value
  - Occasional shocks cause agents to lower their risk estimates
  - This causes them to mistakenly think that recent price innovations are permanent
  - Bid up stock prices, bubble ensues.
  - Eventually, though risk estimates increase, thereby crashing the bubble.