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Alessio Brown, *Christian Merkl and Dennis Snower

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An Incentive Theory of Matching

Alessio J. G. Brown^{a,b}, Christian Merkl^{a,b,c}, and Dennis J. Snower^{a,b,c,d}

^a Kiel Institute for the World Economy, ^b Christian-Albrechts-Universität, Kiel,

^c IZA, ^d CEPR.

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Abstract

This paper presents a theory explaining the labor market matching process through microeconomic incentives. There are heterogeneous variations in the characteristics of workers and jobs, and firms face adjustment costs in responding to these variations. Matches and separations are described through firms' job offer and firing decisions and workers' job acceptance and quit decisions. This approach obviates the need for a matching function. On this theoretical basis, we argue that the matching function is vulnerable to the Lucas critique. Our calibrated model for the U.S. economy can account for important empirical regularities that the conventional matching model cannot.

Keywords: Matching, incentives, adjustment costs, unemployment, employment, quits, firing, job offers, job acceptance.

JEL classification: E24, E32, J63, J64

1 Introduction

The literature on search and matching in the labor market rests heavily on the assumption of a stable matching function (Mortensen and Pissarides, 1994). According to Pissarides (2000, p. 3-4), the matching function aims to summarize “heterogeneities, frictions and information imperfections” and represent “the implications of the costly trading process without the need to make the heterogeneities and the other features that give rise to it explicit.” This paper provides a simple analytical framework that makes the above heterogeneities and frictions explicit, and describes the matching process as the outcome of optimizing behavior by firms and workers. In this context, we show that the matching process is affected by macroeconomic and policy parameters. On a conceptual level, this implies that the relation

between new hires (rather than contacts) and match inputs (such as unemployment and vacancies) - a relation commonly called the matching function - is not stable with respect to policy changes and macroeconomic variations. In short, this matching function runs afoul of the Lucas critique.

The basic intuition underlying this result is straightforward. Although it is often claimed that the matching function is analogous to a production function, an important difference stands out. A firm's production function captures the portfolio of available technologies, and these are indeed often invariant with respect to many policy and macroeconomic variations. By contrast, a matching function summarizes the market activity generated by the decisions of firms and workers, responding to their individual incentives to create jobs, and these incentives are in general *not* invariant with respect to policy and macroeconomic variations. On the contrary, policy changes and macroeconomic shocks usually affect firms' incentives to offer jobs and workers' incentives to accept them. Thus, the relation between new hires and match inputs is mediated by these policy and macroeconomic variations. On this account, the matching function may be expected to change when these variations occur. This calls into question the usefulness of the conventional matching function for forecasting or policy analysis.

In this respect, the matching function faces a difficulty analogous to that of the adaptive expectations hypothesis, which sought to predict expectations without reference to actual policy and macroeconomic variations. As the rational expectations hypothesis took account of how these variations influence expectations and generated new predictions regarding the effectiveness of monetary policy, so our analysis seeks to consider the influence of these variations on new hires and thereby helps account of stylized facts that the conventional matching models have difficulty explaining.

Our account of labor market matches may be called an *incentive theory of matching*, since it explains the matching process in terms of the incentives and economic agents face. This theory provides a different view of matching than that presented by a matching function. Whereas the matching function depicts matches as the output of a "matching technology" that mechanically pairs unemployed workers and vacant firms, the incentive theory explains the matching probability in terms of the firm's job offer incentives and the worker's job acceptance incentives. Our theory explains job separations along analogous lines. In particular, the separation probability is derived from the firm's firing incentives and the worker's quit incentives.

We calibrate an incentive model for the U.S. economy and show that it can account for some important empirical regularities that the conventional matching model cannot. First, our model generates labor market volatilities that are close to what can be found in the

empirical data, specifically for the unemployment rate, the job finding rate and the separation rate. This is remarkable, as we do not rely on any type of real wage rigidity. Instead, our calibration permits us to replicate the stylized fact that wages are as volatile as productivity (see, for example, Hornstein et al., 2005). The standard calibration of the conventional matching model¹ (with exogenous or endogenous separations) is unable to generate these high volatilities of labor market variables (see Shimer, 2005). Second, it generates a strong negative correlation between the job finding rate and the unemployment rate. And third, it can account for a strong negative correlation between job creation and job destruction. The standard calibrations of the matching model, with endogenous job destruction (see Krause and Lubik, 2007), cannot account for these last two stylized facts.²

Intuitively, the reason our model is more successful than the conventional matching model at replicating these stylized facts is that macroeconomic shocks are propagated differently. In the conventional matching model, the employment effect of a change in aggregate productivity depends on the change in new hires generated by a stable, exogenously given matching function. In our incentive model, the adjustments are made on a different margin. Since the agents in our model face heterogeneous match-specific shocks, a change in aggregate productivity affects the range of match-specific shocks over which firms are willing to make job offers and workers are willing to accept these offers. Since aggregate productive shocks are autocorrelated, they can have a substantial leverage effect on the expected present value of profit generated by newly hired workers and incumbent workers, and thereby a strong effect on the hiring and separation thresholds. This helps to understand why our incentive model is more successful than the conventional matching model in generating the observed high volatilities of the unemployment rate, the job finding rate. The other stylized facts can be understood intuitively along the same lines.

The rest of this paper is organized as follows. Section 2 covers the conceptual issues underlying our approach to matching. Section 3 sets the stage by presenting a particularly simple incentive model of matching. Section 4 proves that the simple incentive model cannot be replicated by a standard matching model. Section 5 presents an extended incentive model. Section 6 discusses the calibration strategy. Section 7 presents the numerical results

¹The “standard” calibration of the model excludes rigid wages and small surplus calibrations. Although the rigid wage version of the search and matching model can also generate higher volatilities (Hall, 2005), it implies that counterfactual prediction that wages are acyclical. Thus we do not make this assumption here. We also do not rely on Hagedorn and Manvoskii’s (2008) small surplus calibration, in which the average unemployed worker is basically indifferent between working and not working. In the calibrated version of our model, the current period’s utility of an average unemployed is only about 60% of the utility of an employed.

²The search and matching model with exogenous job destruction actually has a strong negative correlation between the job finding rate and the unemployment (see Shimer, 2005). However, there is an intensive debate in the literature whether separations are exogenous or not (see, for example, Hall, 2006, and Fujita and Ramey, 2009, for opposing views). Separations are endogenous in our analysis.

and Section 8 concludes.

2 Conceptual Issues

Conceptually, the process of job matching and separation may be decomposed into the following stages (as illustrated in Figure 1):

- Stage 1: *Potential searchers* decide on their speed of arrival in the labor market, for a given period of analysis. They thereby turn into *active searchers*. (The relevant friction is an arrival cost.)
- Stage 2: Some of the active searchers make contact with potential employers (e.g., through interviews) and thereby become *applicants*. (The relevant friction is a search cost.)
- Stage 3: The applicants and their potential employers experience match-specific shocks and evaluate the potential expected surplus from the match. Applicants whose surplus is positive turn into *entrants* (new hires). (The relevant friction is a hiring cost.) The entrants turn into *incumbents* at the end of the period of analysis.
- Stage 4: The incumbents and their employers experience further match-specific shocks³ and evaluate the potential expected surplus from the continuation of the match. Incumbents whose surplus is positive are retained. (The relevant friction is a firing cost.)

In the conventional search literature, the first activity (the arrival rate) is assumed exogenously given, while the matching function may play a role in describing the second activity (making contact) and the third activity (evaluating entrant match suitability). The separation rate is the outcome of the fourth activity (evaluating incumbent match suitability); it is assumed exogenous in some search models (e.g., Pissarides, 2000, chapter 1, Hagedorn and Manovskii, 2008, Hall, 2005, Shimer, 2005) and endogenous in others (e.g., den Haan, Ramey and Watson, 2000, Krause and Lubik, 2007, Fujita and Ramey, 2009).

In this context, the matching function can be interpreted in two ways. In the standard interpretation, it covers the second and third activities above, explaining how a given number of job searchers and vacancies leads to a specific amount of new hires. This is the straightforward meaning of Pissarides (2000, p. 3-4), quoted above, and accords with many other explanations of the matching function, such as that of Petrongolo and Pissarides (2001, p.

³In this paper, we assume for simplicity that entrants and incumbents are hit by the same match-specific shock.

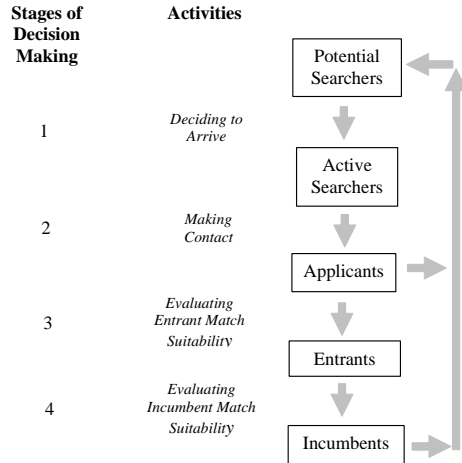


Figure 1: Figure 1

390): "The attraction of the matching function is that it enables the modeling of frictions in otherwise conventional models, with a minimum of added complexity. Frictions derive from information imperfections about potential trading partners, heterogeneities..."

In a second interpretation of the matching function, it covers only the second activity above, namely the process whereby job searchers and vacancies lead to interviews. Under this interpretation, the term "matching function" is a misnomer, since interviews need not lead to matches (i.e. employment relationships); it would be more appropriate to call it a "contact function." This is a natural interpretation of theoretical models that offer microfoundations of the matching by investigating the probability that randomly searching workers and firms (or, more generally, buyers and sellers) find one another, for in this context a homogeneous set of workers simply make contact with a homogeneous set of vacancies (for example, Burdett, Shi and Wright, 2001, Montgomery, 1991, and others).

It is tempting to interpret a recent generation of search models in this vein. Here workers and firms are first matched through a matching function, and then decide whether to continue to sever the contact in response to productivity perturbations. (Examples of such "productivity perturbation models" include Mortensen and Pissarides (1994), den Haan, Ramey and Watson (2000) and others). Some of the discrete-time versions of these models cannot be given the second interpretation (since matches that take place in period t give rise to output and are not broken until period $t + 1$; see, for example, Thomas and Zanetti, 2008, or chapter 7 of Christoffel et al., 2009).

However, some discrete-time versions (where the match may break before becoming productive; see, for example, Gertler and Trigari, 2009, Hagedorn and Manovskii, 2008, Krause and Lubik, 2007, Thomas, 2008) and the continuous-time versions (see, for example, An-

dolfatto, 1996, Costain and Reiter, 2008, den Haan, Ramey and Watson, 2000, Merz, 1995, Shimer, 2005) can (since the match making and breaking occurs at the same instant). Under the second interpretation, workers and vacancies making contact at time t (as described by the matching function) may either form an employment relationship if the current productivity shock is favorable, or split up if the shock is unfavorable.

However, this interpretation does not fit comfortably for the following reasons. First, when these models are calibrated, the calibration relates unemployment and vacancies to new hires, not to contacts (such as interviews). Second, calibrating with respect to contacts would require vast new data sets on formal and informal meetings between searching workers and searching employers and these data sets are not currently available. Thus, the contact-function interpretation is not as yet empirically implementable. Third, these models invariably assume that the proportion of interviews that do not lead to hiring is equal to the proportion of currently employed workers who separate from their jobs. This is surely unrealistic; in practice, interviews may be expected to fail far more frequently than existing employment relationships. In view of these difficulties, it is clear that the first interpretation of the matching function is appropriate for the productivity-perturbation models, i.e., the matching function explains new hires (at time t) and adverse productivity shocks explain separations (at time t).

In sum, the matching function faces a two-pronged challenges. First, if it is interpreted as a contact function, it ceases to be empirically implementable, due to the absence of contact data. And even if it were empirically implementable, it would be necessary to distinguish sharply between contacted workers who turn into new hires and incumbent employees who are retained. Second, if it is interpreted as literal "matching" function - describing how job searchers and vacancies become matched to yield new hires - then we are left with the question whether this matching function is an adequate summary not only of the second activity above (making contact) - as explained in the microfounded models of matching - but of the third activity (evaluating entrant match suitability) as well.

To address this question, we examine whether a stable matching function can reproduce the behavior of optimizing agents evaluating the match suitability of heterogeneous entrants. Accordingly, we will construct a choice-theoretic model with match-specific shocks to firms' profitability and households' utility, where new hires are explained through firms' profit-maximizing decisions to offer jobs and workers' utility-maximizing decisions to accept such job offers. Assuming this choice-theoretic model to be the "true" description of the labor market, we ask whether it is possible to specify a matching function - relating new hires to match inputs (unemployment and vacancies) - that replicates the behavior of the choice-theoretic model, for any values of the model's macroeconomic and policy parameters.

We show that this is not possible in general; for such replication to occur, the matching function generally needs to shift in response to macroeconomic and policy changes. In this sense, the matching function is vulnerable to the Lucas critique.⁴ The upshot of this analysis is that while the matching function may be appropriate in describing the process of making contact (Stage 2) between job searchers and potential jobs – a process of random choice among agents that look homogeneous before the contacts are made – it is not suitable for describing the evaluation of entrant match suitability (Stage 3), and should be replaced by a microfounded model in which new hires are explained in terms of the optimizing decisions of firms and workers in the presence of heterogeneous contacts.⁵

Section 2 constructs a simple model of this sort. Our model differs from the many of the existing productivity-perturbation models in the following respects. First, the match-specific shocks in our model – that explain how many new hires are generated by a given number of contacts – are not just productivity perturbations, but shocks to both firms’ profitability and workers’ disutility of work. The matching rate in our model is not the same as the job offer rate (as in the conventional search models), but depends on both the firms’ job offer rate and workers’ job acceptance rate. Second, separations in our model are also generated by both profitability and utility shocks (which are for simplicity set to the same distribution). The separation rate in our model is not exogenous (as in some conventional search models) or the same as the firing rate (as in others), but depends on both the firms’ firing rate and the households’ quit rate. Third, matches and separations in our model are associated with frictions in the form of hiring and firing costs. (The hiring costs are not to be confused with vacancy posting costs, since the vacancy posting costs are incurred before the contact is made, whereas the hiring costs are incurred after the contact.) Hiring and firing costs drive a wedge between the job-finding and the retention rate, so that the proportion of contacts that lead to new hires is not necessarily equal to the proportion of incumbent workers that are retained.

To keep our analysis in the next section as simple as possible, we focus primarily on Stage-3 activity (evaluating entrant match suitability). For this purpose, we make two

⁴Various authors (e.g. Lagos, 2000) have noted that policies which affect labor market heterogeneities (e.g. retraining programs), frictions (e.g. job counselling) and information imperfections (e.g. job exchanges) may naturally be expected to influence the matching function. In short, there is no reason to believe that the matching function is invariant with respect to labor market policies that are designed to improve the matching process. Our analysis shows, however, that the Lucas critique extends well beyond such policies. Several empirical studies indicate instabilities of the matching function. Often a negative time trend is found when estimating the search and matching function, thus casting doubt on the stability through time (Blanchard and Diamond, 1989, for the United States, and Fahr and Sunde, 2001, 2004, for Germany).

⁵While various other authors have modeled the matching process without resorting to a matching function (e.g. Hall, 1977, Lagos, 2000, Shimer, 2007, and others), our analysis explicitly focuses on two-sided search (i.e. search by both workers and firms at the same time).

simplifying assumptions: (i) the arrival rate of job searchers is an exogenous constant (in Stage 1) and (ii) every searcher is an applicant for one vacancy (in Stage 2). The latter assumption is equivalent to a trivial contact function: $C_t = U_t$, where C_t is the number of contacts (interviews) made in period t and U_t is the number of unemployed job searchers. In this setting it is straightforward to show that the matching function – as summary of Stage-2 and Stage-3 activities – cannot in general reproduce the optimizing decisions of workers and employers, for any values of the macroeconomic and policy variables. It will then be easy to understand why the matching function also cannot replicate the behavior of optimizing agents when the contact function has the standard form $C_t = C(U_t, v_t)$, where v_t are vacancies.

The match-specific heterogeneous shocks, necessitating the evaluation of entrant match suitability, are particularly important in accounting for unemployed workers alongside unfilled jobs. In practice, workers are rarely indistinguishable in terms of all the characteristics relevant for firms' job offer decisions, and jobs are rarely indistinguishable in terms of all the characteristics relevant for workers' job acceptance decisions. Moreover, in practice the firms' activity of screening heterogeneous applicants (Stage 3) is often likely to be far more costly than posting vacancies (Stage 2). Similarly, the workers' activity of evaluating the suitability of alternative workplaces (Stage 3) is often likely to be much costlier than reading help-wanted ads (Stage 2).

Since the match-specific heterogeneous shocks in our model are transient, the resulting evaluation of match suitability may be identified as "search" rather than "mismatch," in terms of the distinction made by Shimer (2007, p. 1074): "According to search theory, unemployed workers have left their old jobs and are actively searching for a new employer. In contrast, this dynamic model of mismatch emphasizes that unemployed workers are attached to an occupation and a geographic location in which jobs are currently scarce. Mismatch is a theory of former steel workers remaining near a closed plant in the hope that it reopens. Search ... is a theory of former steel workers moving to a new city to look for positions as nurses." Thus, the distinction between search and mismatch rests on the persistence of the match-specific shocks and the magnitude of the adjustment costs. If the shocks are transient and the adjustment costs are sufficiently small (as in our analysis), then workers are engaged in search.

In the following we will develop a simple incentive model to show that the matching function cannot reproduce the relation between matches and match inputs (unemployment and vacancies) as generated by a contact function and optimizing Stage-3 decisions. We then extend our model in order to provide the theoretical underpinning of a model that we calibrate and use to explain various stylized facts that are not accounted for in the

conventional search literature.

3 A Simple Incentive Model

To set the stage, we begin by constructing a particularly simple model of the incentive theory of matching, based on heterogeneous match-specific shocks. Our model has the following sequence of labor market decisions. First, vacancies are posted. Second, the realized values of the shocks are revealed. Third, the firms make their hiring and firing decisions and the households make their job acceptance and refusal decisions, taking the wage as given. Unemployed workers search for jobs; employed workers do not.

The purpose of this model is to show that, in this context, the matching function – as a relation between new hires and match inputs (unemployment and vacancies) – is subject to the Lucas critique. To make this point as simply as possible, we make some strong simplifying assumptions. As noted, we assume that job searchers arrive in the labor market at an exogenously constant rate (in Stage 1). Moreover, the length of the period of analysis is such that every searcher finds one vacancy (in Stage 2) per period.

In addition, we assume in this section that the real wage w is exogenously given. (This assumption is relaxed in the next section, where wages are determined through bargaining.) Finally, to provide a maximally transparent comparison of our incentive model and the standard matching model, we assume that workers and firms are myopic (i.e. their rates of time discount are 100%). (This assumption is also relaxed in the next section.)

3.1 The Firm's Behavior

We assume that the profit generated by a particular worker at a particular job is subject to a match-specific random shock ε , which is meant to capture idiosyncratic variations in workers' suitability for the available jobs. For example, workers in a particular skill group and sector may exhibit heterogeneous profitabilities due to random variations in their state of health, levels of concentration, and mobility costs, or to random variations in firms' operating costs, screening, training, and monitoring costs, and so on. The random shock ε is positive and iid across workers, with a stable probability density function $G_\varepsilon(\varepsilon)$, known to the firm.⁶ Let the corresponding cumulative distribution be $C_\varepsilon(\varepsilon)$. In each period of analysis a new value of ε is realized for each worker.

The average productivity of each worker is a , a positive constant. The hiring cost h per worker is also a constant. The hiring cost includes the administrative costs, screening

⁶Our analysis can of course be extended straightforwardly to shocks with AR and MA components.

costs, retraining costs, and relocation costs, as well as the basic instruction, mentoring and on-the-job training costs that are required to integrate the worker in the firm's workforce. The profit generated by an entrant (a newly hired worker) is

$$\pi^E = a - \varepsilon - w - h, \quad (1)$$

where the superscript "E" stands for "entrant" and w is the real wage.

The firm's "job offer incentive" (its payoff from hiring a worker) is the difference between its gross profit⁷ from hiring an entrant worker ($a - w - h$) and its profit from not doing (namely, zero):

$$\nu^E = a - w - h. \quad (2)$$

The firm offers this job to a worker whenever that worker generates positive profit: $\varepsilon < \nu^E$. Thus the job offer rate is

$$\eta = C_\varepsilon(\nu^E). \quad (3)$$

The firm's "retention incentive" (its payoff from retaining a worker) is the difference between its gross profit from retaining a worker is ($a - w$) and the (negative) profit from firing that worker:

$$\nu^I = a - w + f, \quad (4)$$

where the superscript "I" stands for the incumbent employee who has been retained, and f is the firing cost per worker, assumed constant. The firm with a filled job will fire an incumbent worker whenever she generates negative profit: $\varepsilon > \nu^I$. Thus the firing rate is:

$$\phi = 1 - C_\varepsilon(\nu^I). \quad (5)$$

Note that due to the hiring and firing costs, the retention incentive exceeds the job offer incentive ($\nu^I > \nu^E$) and thus the retention rate exceeds the job offer rate ($(1 - \phi) > \eta$).

3.2 The Worker's Behavior

The worker faces a discrete choice of whether or not to work. If she works, her disutility of work effort is e , which is a random variable, which is iid, with a stable probability density function $G_e(e)$, known to the worker. The corresponding cumulative distribution is $C_e(e)$. The random variable captures match-specific heterogeneities in the disagreeability of work, due to such factors as idiosyncratic reactions to particular workplaces. If the worker does

⁷This "gross" profit is the expected profit generated by hiring an unemployed worker, without taking the match-specific shock ε into account.

not work, her utility is b (a constant). Her utility is linear in consumption and work effort.⁸ She consumes all her income. Thus the utility of an employed worker is $\Omega^N = w - e$, and the utility of an unemployed worker is $\Omega^U = b$.

A worker’s “work incentive” (her payoff from choosing to work) is the difference between her gross utility from working (w) and her utility from not working (b):

$$\iota = (w - b). \tag{6}$$

Assuming that $w > b$ and letting $E(e) = 0$, all unemployed workers have an ex ante incentive to seek work.

An unemployed worker will accept a job offer whenever $e < \iota$. This means that the job acceptance rate is

$$\alpha = C_e(\iota). \tag{7}$$

Along the same lines, an employed worker will decide to quit when $e > \iota$. This means that the quit rate is

$$\chi = 1 - C_e(\iota). \tag{8}$$

Note that, for simplicity, we have assumed that the job acceptance rate is identical to the job retention rate ($\alpha = 1 - \chi$). When unemployed workers face costs of adjusting to employment (e.g. buying a car to get to work, or psychic costs of changing one’s daily routine) or when employed workers face costs of adjusting to unemployment (e.g. building networks of friends with potential job contacts, psychic costs of adjusting to joblessness), then the job acceptance rate would fall short of the job retention rate.⁹

3.3 Match and Separation Probabilities

An unemployed worker gets a job when two conditions are fulfilled: (i) she receives a job offer *and* (ii) she accepts that offer. Thus the *match probability* (μ) is the product of the job offer rate (η) and the job acceptance rate (α):

⁸Observe that on the firm’s side, we distinguish between entrants (E) and incumbent workers (I); whereas on the workers’ side, we distinguish between employed (N) and unemployed (U) workers. The rationale for these two distinctions is that the firm can hire two types of workers (entrants and incumbents), whereas the worker can be in two states (employment and unemployment).

⁹Specifically, for example, the unemployed worker’s job acceptance incentive could be expressed as $\iota^U = w - b - \xi^U$, where ξ^U is the cost of adjusting to employment, and the incumbent worker’s job retention incentive could be expressed as $\iota^N = w - b + \xi^N$, where ξ^N is the cost of adjusting to unemployment. Then the job acceptance rate becomes $\alpha = C_e(\iota^U)$, the job retention rate becomes $C_e(\iota^N)$ so that the quit rate becomes $\chi = 1 - C_e(\iota^N)$.

$$\mu = \eta\alpha. \quad (9)$$

An employee separates from her job when at least one of two conditions is satisfied: (i) she is fired *or* (ii) she quits. Thus the *separation probability* is

$$\sigma = \phi + \chi - \phi\chi. \quad (10)$$

3.4 Vacancies

Vacancies are posted before ε is realized. For simplicity, we make the common assumption that each firm has one job, which may be filled or remain vacant.¹⁰ As in the conventional search literature, we assume free entry of firms, so that the number of vacancies is determined by a zero-profit condition. Let v be the number of vacancies, and κ be the cost of posting a vacancy. If $v \geq uL$, then the probability that a vacancy is filled is $(uL/v)\mu$, i.e. the probability of a contact times the probability that the contact leads to a match. The expected profit per match is $(a - w - h - E(\varepsilon))$.

Thus the zero-profit condition for posting vacancies is $E(\pi) = \mu(uL/v)(a - w - h - E(\varepsilon)) - \kappa = 0$, implying that the equilibrium number of vacancies is

$$v^* = (a - w - h - E(\varepsilon)) \frac{\mu u L}{\kappa} \quad (11)$$

If $v < uL$, then all vacancies are filled and the expected profit from posting a vacancy is $E(\pi) = \mu(a - w - h - E(\varepsilon)) - \kappa$. If $E(\pi) < 0$, then no vacancies are posted; whereas if $E(\pi) > 0$, then the firm continues to post vacancies until $v^* = uL$ (for which eq (11) applies).

3.5 The Labor Market Equilibrium

Given that all unemployed people are job searchers and assuming that each job searcher makes one contact per period, the number of unemployed workers who get jobs in period t is μu_{-1} , where u_{-1} is the number of unemployed in the previous period.¹¹ The number of employed people who separate from their jobs in period t is σn_{-1} , where n_{-1} is the number of employed in the previous period.

The change in employment is $\Delta n = n - n_{-1} = \mu u_{-1} - \sigma n_{-1}$. The labor force L is assumed constant and normalized to unity (so that n and u are both levels and rates of employment

¹⁰In the Appendix we present a model in which a firm can post multiple vacancies.

¹¹All other variables (without subscripts) refer to the current period.

and unemployment, respectively). Thus $u = 1 - n$ and equilibrium employment n^* may be described by the following employment equation:

$$n^* = \mu + (1 - \mu - \sigma)n_{-1}^*. \quad (12)$$

Substituting $u^* = 1 - n^*$ into the vacancy equation (11), we obtain the equilibrium number of vacancies. Observe that vacancies play no allocative role in our model: the number of vacancies has no effect on employment and unemployment. The reason is that vacancies do not influence the number of contacts made by a given number of unemployed job searchers, since we have assumed that the number of contacts is equal to the number of job searchers. Instead, vacancies in our model are simply an attention-eliciting device: the greater the number of vacancies that are posted in the economy, the lower is the probability that they will be filled by a given number of job searchers.

4 Is the Matching-Function Subject to the Lucas Critique?

We now juxtapose the model above with its matching-function counterpart in order to investigate whether the matching function runs afoul of the Lucas critique. For this purpose, let us assume that the incentive model above describes the real world, and then let us ask whether the behavior of this model can be replicated by a corresponding model containing a matching function. We will show that such replication cannot occur unless the matching function changes whenever the underlying parameters of the model change. These parameter changes include macroeconomic variables (such as productivity, a) and policy variables (such as unemployment benefits, underlying the parameter b). Thus, in this analytical context, the matching function runs afoul of the Lucas critique: policy analysis and comparative static prediction on the basis of a stable matching function would yield misleading results.

Naturally, the incentive model above is extremely simple, but it is precisely this simplicity that allows us to bring the Lucas critique of the matching function into sharp relief. The same critique can be formulated with respect to more complicated models (such as the one in the next section), since the underlying idea is quite general: For any given matching function - specified independently of the optimizing decisions relevant to the matching process - it is always possible to construct a microfounded macro model that systematically fools this matching function. In this sense, the difficulty of the matching functions is analogous to that of expectation-generating mechanisms in traditional macro models that were incompatible with rational expectations.

4.1 The Matching-Function Representation

We now specify the matching-function counterpart to the incentive model above. Let the matching function be

$$x = x(u, v), \quad (13)$$

where u is the unemployment rate and v is the vacancy rate. This function satisfies the standard conditions: $x_i > 0$, $x_{ii} < 0$, $i = u, v$; $x(u, 0) = x(0, v) = 0$; and there are constant returns to scale: $gx(u, v) = x(gu, gv)$ where g is a positive constant.

Let $\theta = v/u$ denote labor market tightness, so that $q(\theta) = x(u/v, 1)$ is the probability that a job is matched with a worker, and $\theta q(\theta)$ is the probability that a worker is matched by a job. Along the lines of the simple labor market matching models, we assume that jobs are destroyed at an exogenous rate λ , $0 < \lambda < 1$. Then the change in the employment rate is¹² $\Delta n = \theta q(\theta)(1 - n_{-1}) - \lambda n_{-1}$, implying the following employment dynamics equation:

$$n = \theta q(\theta) + (1 - \theta q(\theta) - \lambda) n_{-1}. \quad (14)$$

Vacancies are posted until the expected profit is reduced to zero: $a - w = \frac{\kappa}{q(\theta)}$, where κ is a vacancy posting cost, $\kappa/q(\theta)$ is the expected vacancy posting cost per worker. Expressing this zero-profit condition in terms of labor market tightness:

$$\theta = g \left(\frac{\kappa}{a - w} \right), \quad (15)$$

where $g = q^{-1}$.

The equilibrium employment rate n is obtained by substituting the zero-profit condition (15) into the employment dynamics equation (14).

4.2 Equivalence Condition

In order for the two models to be comparable, let the exogenous wage w be identical in both models and suppose that the separation rate σ in the incentive model is a constant equal to the job destruction rate λ in the conventional matching model. Then the two models are observationally equivalent when $\theta q(\theta) + (1 - \theta q(\theta) - \sigma) n_{t-1} = \mu + (1 - \mu - \sigma) n_{t-1}$, so that

$$\theta q(\theta) = \mu, \quad (16)$$

¹²To keep this model comparable with our the simple incentive model above, we assume (without loss of generality) the same timing in both models. Matches are not destroyed in the match period and they become immediately productive.

which we call the “equivalence condition.” It may be expressed as

$$\frac{\kappa}{a-w}g\left(\frac{\kappa}{a-w}\right) = C_\varepsilon(a-w-h)C_e(w-b). \quad (17)$$

To examine whether the matching function is vulnerable to the Lucas critique, we ask the following question: for given cumulative distributions C_ε and C_e , can a matching function g be found, so that the condition (17) holds for any values of the parameters which the incentive model and the conventional matching model have in common? To answer this question, we differentiate condition (17) with respect to productivity a ,

$$-\frac{\kappa}{(a-w)^2}\left[\frac{\kappa}{a-w}g' + g\right] = C'_\varepsilon C_e. \quad (18)$$

Differentiating with respect to w ,

$$\frac{\kappa}{(a-w)^2}\left[\frac{\kappa}{a-w}g' + g\right] = -C'_\varepsilon C_e + C_\varepsilon C'_e. \quad (19)$$

Differentiating with respect to κ ,

$$\frac{\kappa}{a-w}g'\left(\frac{\kappa}{a-w}\right) + g\left(\frac{\kappa}{a-w}\right) = 0 \quad (20)$$

Conditions (18) and (19) are mutually exclusive, unless the slope of the cumulative distribution of e is $C'_e = 0$. The latter implies that the underlying density is $G_e = 0$. This occurs when there exist no households with a marginal disutility of effort e over the relevant range. (It is on this account that the number of households that accept jobs is not affected by $(w-b)$.) A distribution of e with zero mass is indeed a special case; it amounts to excluding the possibility of heterogeneous workers in our model.

Conditions (18) and (20) are mutually exclusive, unless the slope of the cumulative distribution of ε is $C'_\varepsilon = 0$, which implies that the underlying density is $G_\varepsilon = 0$. This occurs when there exist no jobs with workplace heterogeneities ε over the relevant range, i.e. the possibility of heterogeneous profitabilities is excluded.

In short, there exists no functional form for g such that condition (17) always holds - for any given cumulative distributions C_ε and C_e , and for any values of the parameters common to the two models - unless heterogeneities on the firm and households side are absent. This is another way of saying that the matching function is not observationally equivalent to an explicit description of firm and household choices under heterogeneities.

Thus the standard matching model cannot reproduce the labor market dynamics of the incentive model above. This non-equivalence is not a special case to be ascribed to the

particular specification of the incentive model. It is easy to see that the reasoning above is applicable to a broad family of models. The source of the non-equivalence is analogous to the non-equivalence of adaptive-expectations and rational-expectations macro models. Adaptive-expectations models were unable to reproduce the dynamics of rational-expectations models because, for any given function specifying adaptive expectations, it is always possible to find a hypothetically “true” stochastic generating process which produces predictable errors, that is, errors not reconcilable with rational expectations. Along the same lines, the comparison above makes clear that for any given matching function, it is always possible to find a hypothetically “true” model of the underlying heterogeneities and frictions which produces labor market dynamics that cannot be replicated through the matching function. Just as an expectations generating mechanism that is specified a priori (independently of the underlying macro model) is not a reliable tool for investigating the influence of macro policy, so a matching function that is specified a priori is also not a reliable tool to explore the influence of labor market policy. The same can be said regarding the influence of other macro and labor parameters.

Alternatively, we can say that the matching function is not stable with respect to the parameters whose influence the matching models are meant to analyze. If the incentive model above is assumed to be the “true” model of the labor market, then the standard matching model can reproduce the “true” employment effects of variations in all the relevant parameters - the wage w , productivity a , the hiring cost h , or the leisure utility b - *only* if we assume that the matching function is modified whenever these parameters are changed. This instability of the matching function makes it an inappropriate tool for investigating the effectiveness of policy changes or macroeconomic fluctuations.

Although the simple model above is useful to examine why the matching function is subject to the Lucas critique, we now need to relax several restrictive assumptions of the incentive model above - that households and firms are myopic, wages are exogenous, and productivity is constant - in order to examine the relative performance of the incentive model and the standard matching model in accounting for well-known stylized facts. In the context of conventional calibrations, we will show that the incentive model fares better than the standard matching model in reproducing the volatilities of major labor market variables.

5 A Dynamic Incentive Model

We now extend the simple model above by

- including aggregate risk: the average aggregate productivity parameter a is now subject to random productivity shocks;

- allowing for rates of time discount that are less than 100%, so that workers and firms become intertemporal optimizers and
- introducing wage determination through bargaining.

The first extension enables us to simulate productivity shocks as done in Hall (2005), Shimer (2005) and numerous other papers and to make our framework quantitatively comparable to the matching theory. The second and third extensions provide a richer depiction of the determinants of employment and wages.

For simplicity, we will not consider vacancies in our extended model. As noted, they fulfill no allocative role in our model. Beyond that, it is worth noting that vacancy data is the Achilles' heel of conventional empirical matching models. For long time series, we have only a very rough proxy for U.S. vacancies, namely, the Conference Board help-wanted advertising index, measuring the number of help-wanted ads in 51 major newspapers. Over the past decade, this index shows a clear downward trend (adjusting for the business cycle), which may well be due to internet advertising. Although an internet advertising index exists, it is far from clear how this index can be made comparable to the newspaper index. Moreover, while the Conference Board advertising index and the JOLTS survey on vacancies exhibit similar dynamics for the limited sample periods in which comparable data sets are available (Shimer (2005)), it appears, surprisingly, that the number of vacancies (as defined by the JOLTS survey) is consistently and substantially smaller than the number of new hires! There are two obvious reasons why this should be so, both highlighting weaknesses of vacancies data: (i) Only a fraction of the jobs that get filled are preceded by vacancy postings. (The matching function has nothing to say about the many hires that occur without formal advertising.) (ii) The JOLTS survey, like all other surveys, ignores high-frequency vacancy movements. In particular, JOLTS measures end-of-month reported job openings, not job openings that get filled before the month is over. Overall, such considerations indicate that vacancy data is much less reliable than the other data (e.g. unemployment rates, productivity) used in conventional empirical matching models. On account of this as well as analytical simplicity, our incentive model will not cover vacancies.

In this context, the new sequence of decisions may be summarized as follows. First, the aggregate productivity shock and the idiosyncratic shocks are revealed. Second, the wage is set through bargaining. Third, the firms make their hiring and firing decisions and the households make their job acceptance and refusal decisions, taking the wage and the realization of the aggregate and idiosyncratic shocks as given.

5.1 The Firm's Behavior

Since the firm is not myopic in this model, its hiring and firing decisions depend on its expected profits not only in the current time period, but also in future time periods.

5.1.1 The Firing Decision

The expected present value of profit generated by an incumbent employee, after the random profitability term ε_t is observed, is¹³

$$E_t(\pi_t^I) = (a_t - w_t - \varepsilon_t) + \delta E_t(\pi_{t+1}^I), \quad (21)$$

where δ is the time discount factor, a_t is the incumbent employee's productivity, and

$$E_t(\pi_{t+1}^I) = E_t[(1 - \sigma_{t+1})(a_{t+1} - w_{t+1} - E_t(\varepsilon_{t+1} | (\varepsilon_{t+1} < \nu_t^I)) + \delta \pi_{t+2}^I) - \phi_{t+1}f]. \quad (22)$$

$E(\varepsilon | \varepsilon < \nu_t^I)$ is the expectation of the random term ε , conditional on this shock falling short of the *incumbent employee's retention incentive* ν_t^I , which is defined as

$$\nu_t^I = a_t - w_t + \delta E_t(\pi_{t+1}^I) + f, \quad (23)$$

i.e. the retention incentive is the difference between the gross expected profit from retaining the employed worker ($a_t - w_t + \delta E_t(\pi_{t+1}^I)$) and the expected profit from firing her ($-f$).

An incumbent worker is fired in period t when the realized value of the random cost ε_t is greater than the incumbent worker employment incentive: $\varepsilon_t > \nu_t^I$. Since the cumulative distribution of ε is $C_\varepsilon(\nu_t^I)$, the employed worker's firing rate is

$$\phi_t = 1 - C_\varepsilon(\nu_t^I). \quad (24)$$

5.1.2 The Job Offer Decision

The expected present value of profit generated by an entrant, after the random cost ε_t has been observed, is

$$E_t(\pi_t^E) = a_t - w_t - \varepsilon_t - h + \delta E_t(\pi_{t+1}^I). \quad (25)$$

¹³In the first period, profit is $(a_t - w_t - \varepsilon_t)$; in the second period, the worker is retained with probability $(1 - \phi_t)$ and then generates an expected profit of $a_t - w_t$, and the worker is fired with a probability of ϕ_t and then generates a firing cost of f_t ; and so on.

We define the firm's expected *job offer incentive* ν_t^E as the difference between the gross expected profit from a hired worker ($a_t - w_t - h + \delta E_t(\pi_{t+1}^I)$) and the profit from not hiring him (i.e. zero):

$$\nu_t^E = a_t - w_t - h + \delta E_t(\pi_{t+1}^I) \quad (26)$$

A job is offered when $\nu_t^E > \varepsilon_t$. Thus the job offer rate is

$$\eta_t = C_\varepsilon(\nu_t^E). \quad (27)$$

5.2 The Worker's Behavior

The incumbent worker's expected present value of utility ex post (once the realized value of the disutility shock e_t is known) is

$$E_t(\Omega_t^N) = w_t - e_t + \delta E_t((1 - \sigma_{t+1})\Omega_{t+1}^N + \sigma_{t+1}\Omega_{t+1}^U), \quad (28)$$

where $E_t(\Omega_{t+1}^N)$ is the expected present value of utility of the following period (before the realized value of the shock e_t is known):

$$E_t(\Omega_{t+1}^N) = E_t(w_{t+1} - E_t(e_{t+1}|e_{t+1} < \iota_{t+1}) + \delta((1 - \sigma_{t+2})\Omega_{t+2}^N + \sigma_{t+2}\Omega_{t+2}^U)). \quad (29)$$

The expected present value utility from unemployment is

$$E_t(\Omega_t^U) = b_t + \delta E_t(\mu_{t+1}\Omega_{t+1}^N + (1 - \mu_{t+1})\Omega_{t+1}^U). \quad (30)$$

An unemployed's expected "*work incentive*" ι_t (the incentive for an unemployed to accept work) is the expected difference between the gross¹⁴ present value from working $E_t(\tilde{\Omega}_t^N) = w_t + \delta E_t((1 - \sigma_{t+1})\Omega_{t+1}^N + \sigma_{t+1}\Omega_{t+1}^U)$ and the present value from not working $E_t(\Omega_t^U)$ in the current period:

$$\iota_t = E_t(\tilde{\Omega}_t^N - \Omega_t^U). \quad (31)$$

Thus the unemployed accepts a job offer when $e_t < E_t(\tilde{\Omega}_t^N - \Omega_t^U)$, so that $e_t < \iota_t$. Consequently, the job acceptance rate is

$$\alpha_t = C_e(\iota_t). \quad (32)$$

¹⁴The employed worker's "gross" expected present value from working is the employed worker's expected present value of utility without taking the utility shock into account.

The incumbent worker decides to quit his job when the present value of becoming unemployed exceeds the present value of remaining employed ($E_t(\tilde{\Omega}_t^N) - e_t < E_t(\Omega_t^U)$), so that his expected work incentive is lower than the utility cost $e_t > E_t(\tilde{\Omega}_t^N - \Omega_t^U) = \iota_t$. Thus the quit rate is

$$\chi_t = 1 - C_e(\iota_t). \quad (33)$$

5.3 Employment

As in the previous model, the match probability is

$$\mu_t = \eta_t \alpha_t, \quad (34)$$

and the separation probability is

$$\sigma_t = \phi_t + \chi_t - \phi_t \chi_t, \quad (35)$$

and the associated employment dynamics equation is

$$n_t = \mu_t + (1 - \sigma_t - \mu_t) n_{t-1} \quad (36)$$

where the employment persistence parameter ($1 - \sigma_t - \mu_t$) depends inversely on the match probability μ_t and the separation probability σ_t . An alternative interpretation of the persistence parameter is given by

$$1 - \sigma_t - \mu_t = (1 - \phi_t)(1 - \chi_t) - \eta_t \alpha_t, \quad (37)$$

where $(1 - \phi_t)(1 - \chi_t)$, the product of the incumbents' retention rate and staying rate, is the incumbents' survival rate. Thus the persistence parameter is the difference between the incumbents' survival rate and the unemployed workers' match probability.

5.4 Wage Determination

We now endogenize the real wage through bargaining. The conventional matching models assume that the real wage is the outcome of Nash bargaining, which takes place after the match has been made. This sequence of decisions is conceptually problematic, particularly when match productivities are heterogeneous. Should not the firms' and workers' incentives to match depend on the wage offered? If workers and jobs differ in terms of their productivities, will not a change in the wage lead to a change in the number of matches that are

productive? In practice, of course, we don't find workers and firms agreeing to match before the terms of the employment contract have been set.

This difficulty is easy to overlook in the conventional matching models, where matches are generated mechanically through a matching function, all matches generate a bargaining surplus, and this bargaining surplus is shared by the worker and the firm through the subsequent wage negotiation. But once the matching process is endogenized in terms of the worker's and firm's incentive to match - as is done in our incentive model - the difficulty comes into sharp relief. Then we see that the match probability depends on the firm's job offer rate and the worker's job acceptance rate, and these rates in turn depend on the wage. Similarly, the separation probability depends on the firm's firing rate and the worker's quit rate, and these rates are also wage-dependent. In this context, it is clear that the number of matches made and destroyed per period of time cannot be determined without prior knowledge of the wage.

On this account, we assume here that wage bargaining takes place before the job offer, acceptance, firing and quit decisions are made. Our aim is to formulate a wage determination model that is (i) simple and tractable, (ii) comparable to the wage bargaining process in the conventional matching models (with the exception of the timing issue above) and (iii) able to reproduce the stylized fact that wages are as volatile as productivity. For this purpose, we let the incumbent workers and entrants receive the same wage w_t ,¹⁵ determined through Nash bargaining between the firm and its median incumbent worker. The median worker faces no risk of dismissal, as he is at the middle of the ε distribution. These assumptions satisfy the three aims above, because (i) they simplify the analysis by allowing the employment rate to depend on the wage, but not vice versa, (ii) the Nash bargaining between the firm and the median incumbent is comparable to the wage bargaining in the conventional matching models, and (iii) the negotiated wage turns out to be as volatile as productivity.

Needless to say, other models of wage negotiations could be incorporated into our analysis (e.g. individualistic wage bargaining, monopoly union wage setting, separate wage negotiations for incumbents and entrants, etc.), but we do not do so here since they would substantially complicate the model,¹⁶ without affecting the main points of our analysis, namely, that

¹⁵This assumption also implies that an increase in wages leads to a fall in employment. This employment effect can of course also be generated when incumbent workers and entrants have different wages. For example, Lindbeck and Snower (2001) provide a variety of reasons why entrants do not receive their reservation wage and thus a rise in incumbent workers' wages is not met a countervailing fall in entrant wages, and thus a rise in incumbent workers' wage lead to a fall in employment. In the context of a Markov model, Diaz-Vazquez and Snower (2003) show that incumbent workers' wages are inversely related to aggregate employment even when entrants receive their reservation wages.

¹⁶Wages would depend on the time path of employment, while employment depends on the time path of wages.

the matching and separation rates can be determined endogenously through the job offer, job acceptance, firing and quit decisions, and that these decisions are not replicable through a stable matching function.

The wage bargain takes place in each period of analysis. In the current period t , under bargaining agreement, the median incumbent worker receives the wage w_t incurs effort cost e^M and the firm receives the expected profit $(a_t - w_t - \varepsilon^M)$ in each period t . Thus the expected present value of the median incumbent worker's utility $E(\Omega_t^M)$ under bargaining agreement is

$$E_t(\Omega_t^M) = w_t - e^M + \delta E_t \left((1 - \sigma_{t+1}) \Omega_{t+1}^N + \sigma_{t+1} \Omega_{t+1}^U \right). \quad (38)$$

The expected present value of firm's returns under bargaining agreement are

$$E_t(\Pi_t^M) = (a_t - w_t - \varepsilon^M) + \delta E_t \left((1 - \sigma_{t+1}) \Pi_{t+1}^N - \phi_{t+1} f \right). \quad (39)$$

Under disagreement in bargaining, the incumbent worker's fallback income is d , which can be conceived as financial support from family and friends, strike pay out of a union fund, or other forms of support. The firm's fallback profit is $-z$, a constant. Assuming that disagreement in the current period does not affect future returns, the present value of utility under disagreement for the incumbent worker is

$$E \left(\tilde{\Omega}_t^M \right) = d + \delta E_t \left((1 - \sigma_{t+1}) \Omega_{t+1}^N + \sigma_{t+1} \Omega_{t+1}^U \right), \quad (40)$$

and the present value of profit under disagreement for the firm is

$$E \left(\tilde{\Pi}_t^M \right) = -z + \delta E_t \left((1 - \sigma_{t+1}) \Pi_{t+1}^N - \phi_{t+1} f \right). \quad (41)$$

The incumbent worker's bargaining surplus is

$$\begin{aligned} E_t(\Omega_t^M) - E_t \left(\tilde{\Omega}_t^M \right) &= w_t - e^M + \delta E_t \left((1 - \sigma_{t+1}) \Omega_{t+1}^N + \sigma_{t+1} \Omega_{t+1}^U \right) \\ &\quad - d - \delta E_t \left((1 - \sigma_{t+1}) \Omega_{t+1}^N + \sigma_{t+1} \Omega_{t+1}^U \right) \\ &= w_t - d - e^M, \end{aligned} \quad (42)$$

and the firm's surplus is

$$\begin{aligned}
E_t(\Pi_t^M) - E_t(\tilde{\Pi}_t^M) &= (a - w_t - \varepsilon^M) + \delta E_t((1 - \sigma_{t+1}) \Pi_t^N - \phi_{t+1} f) - \\
&E_t(-z + \delta((1 - \sigma_{t+1}) \Pi_t^N - \phi_{t+1} f)) \\
&= a_t - w_t - \varepsilon^M + z.
\end{aligned} \tag{43}$$

The negotiated wage maximizes the Nash product (Λ):

$$\Lambda = (w_t - e^M - d)^\gamma (a_t - w_t + z - \varepsilon^M)^{1-\gamma}. \tag{44}$$

Thus the negotiated wage is

$$w_t = \gamma (a_t + z - \varepsilon^M) + (1 - \gamma) (e^M + d), \tag{45}$$

where γ represents the bargaining strength of the incumbent worker relative to the firm.

5.5 The Labor Market Equilibrium

The labor market equilibrium is the solution of the system comprising the following equations:

- *Incentives:* the incumbent worker retention incentive ν_t^I (eq. 23), the job offer incentive ν_t^E (eq. 26), the work incentive ι_t (eq. 31).
- *Employment decisions:* the firing rate ϕ_t (eq. 24), and the job offer rate η_t (eq. 27).
- *Work decisions:* the job acceptance rate α_t (eq. 32) and the quit rate χ_t (eq. 33).
- *Match and separation probabilities:* the match probability μ_t (eq. 34) and the separation probability σ_t (eq. 35).
- *Employment and wage determination:* the employment level N_t (eq. 36) and the negotiated wage w_t (eq. 45).

6 Calibration

We now calibrate our incentive model for the US economy. The calibration is done on a monthly basis. The simulation results are aggregated to quarterly frequency to make them comparable to the empirical data, as for example in Shimer (2005). For the discount factor

$\delta = \frac{1}{1+r}$ we apply the real interest rate $r = 1.04^{1/12} - 1$. We normalize the average productivity (a) to 1. As in Hall (2005) and Shimer (2005), we set b by applying a replacement rate of $\beta = 40\%$ of the wage. For simplicity, we set $d = b$. As commonly found in the literature we adopt a bargaining power parameter γ of 0.5.

Vacancy posting costs are usually set to around 30 percent of the quarterly productivity in the conventional matching model calibrations. To make our calibration as comparable as possible to conventional ones, we divide this number by the typical quarterly worker job finding rate of 0.7 (see, e.g., Krause and Lubik, 2007 and den Haan et al., 2000) to obtain the hiring costs, h . This gives us a value of 43 percent of the quarterly productivity or roughly 130 percent of the monthly productivity.

The literature does not provide reliable direct estimates of the magnitude of US firing costs. Thus we assess these costs indirectly. For this purpose, note that Belot et al. (2007) provide index measures of employment protection for regular jobs in the US and UK, and that Bentolila and Bertola (1990) provide estimates of the average magnitude of UK firing costs on a yearly basis.¹⁷ Assuming that the index measures of employment protection are proportional to the estimates of the magnitude of firing costs, we multiply the magnitude of the UK firing costs by the ratio of the US to the UK employment protection indexes to derive a rough estimate of the magnitude of US firing costs. Accordingly, the magnitude of monthly US firing costs, relative to productivity, is 0.08. The same exercise based on other industrialized countries (France, Germany and Italy), however, yields higher estimates of US firing costs. Thus we choose a value of 0.1 for our baseline calibration, but provide a robustness analysis for other values in Appendix B.¹⁸ For simplicity, we set the firm's fallback profit $-z$ equal to $-f$.¹⁹

We assume that the random profitability term ε and the utility shock e have cumulative distributions given by logistic functions with scale factors s_ε and s_e and expected values $\bar{\varepsilon}$ and \bar{e} , respectively.²⁰ We calibrate our model such that it replicates the stylized fact that wages are as volatile as productivity.²¹ This is achieved by setting $\bar{e} = 0.19$. Thereby our calibration excludes the possibility that our results are driven by real wage rigidity.

After having set all our other parameter values, we are left with three free distributional parameters ($\bar{\varepsilon}$, s_e , s_ε) to replicate three steady state labor market flow rates (α , η , and ϕ). Thus, our hands are bound with respect to the distributional parameters and we have no

¹⁷We take averages over the time periods provided by these authors.

¹⁸Specifically, we provide simulation results for firing costs calculated relative to the UK, $f = 0.08$, and as an upper bound we choose $f = 0.2$.

¹⁹Here we implicitly assume that during disagreement the incumbent worker imposes the maximal cost on the firm short of inducing dismissal.

²⁰The cumulative logistic distribution is very close to the cumulative normal distribution.

²¹See Hornstein et al. (2005).

further degrees of freedom.

The distributional parameters are chosen to replicate the following steady state values. The match probability μ , which is the probability for a worker to find a new job within one period, is calibrated to 45%²², as in Shimer (2005) and Hagedorn and Manvoskii (2008). The unemployment rate u is calibrated to 12%.²³ According to our employment dynamics equation (12) steady state unemployment is $u = \frac{\mu}{\mu+\sigma}$ which implies a separation rate of 6.14%. Based on Hall (2006), who shows that fires and quits have approximately the same share in separation, we assume firings to account for 50% of the separations, namely $\phi = 3.1\%$. Eq. (35) then yields the quit rate of $\chi = 3.2\%$. Since α is equal to $1 - \chi$, the job acceptance rate is set at 96.8%. Recalling that $\mu = \alpha\eta$, we find that the resulting job offer rate η is 46.5%.

We normalize the autocorrelation (ρ_a) of the aggregate productivity shock and normalize the standard error such that we obtain the empirical values for the autocorrelation and the volatility of productivity in the model simulation below. Table 2 summarizes our calibrated parameter values.

Variable	In Words	Steady State Value
u	unemployment rate	0.120
μ	match probability	0.450
η	hiring/job offer rate	0.465
σ	separation rate	0.061
ϕ	firing rate	0.031
$\chi = 1 - \alpha$	job quit rate	0.032

Table 1: Steady State Values

²²Note: In our model the worker finding rate (i.e., the probability of a firm to find a new worker) and the job finding rate (i.e., the probability of a worker to find a new) are the same.

²³This value also considers potential participants in the labor market such as discouraged workers and workers loosely attached to the labor force, see Krause and Lubik (2007) and den Haan et al. (2000).

Parameter	In Words	Value
a	productivity	1
β	replacement rate $\frac{b}{w}, \frac{d}{w}$	0.4
f	firing cost	0.1
h	hiring cost	1.3
γ	workers' bargaining strength	0.5
r	discount factor	0.997
$-z$	firm's fallback profit	-0.1
\bar{e}	average value of leisure	0.17
$\bar{\varepsilon}$	average operating costs	0.465
s_ε	scale factor of the cumulative distribution of ε_t	0.390
s_e	scale factor of the cumulative distribution of e_t	0.078
ρ_a	autocorrelation of the aggregate productivity shock	0.975
ϖ_a	standard error of the aggregate productivity shock	0.007

Table 2: Parameter Values (Rounded to the Third Decimal)

7 Description of Results

7.1 Labor Market Volatilities

Costain and Reiter (2008) and Shimer (2005) show that the conventional calibration of the matching model is unable to replicate the volatility of the job finding rate, the unemployment rate, and other labor market variables in response to productivity shocks. Table 3 shows that the empirical volatilities for the United States (from 1951-2003, HP filtered data with smoothing parameter 100,000, as calculated by Shimer) are far greater than the corresponding volatilities in response to productivity shocks, as generated by the simulation of the conventional matching model (in its standard calibration, as calculated by Shimer).

	U. Rate	Match. Rate	Sep. Rate	Product.
Empirical Volatilities by Shimer (2005), from 1951-2003				
Standard deviation	0.19	0.12	0.08	0.02
Relative to productivity	9.5	5.9	3.8	1
Quarterly autocorrelation	0.94	0.91	0.73	0.88
Volatilities by Shimer's (2005) Search and Matching Model				
Standard deviation	0.01	0.01	-	0.02
Relative to productivity	0.5	0.5	-	1
Quarterly autocorrelation	0.94	0.88	-	0.88

Table 3: Empirical Volatilities and Volatilities generated by the Search and Matching Model from Shimer (2005).

To compare our model with the conventional matching theory, we use our baseline calibration (with robustness checks in the Appendix B) to simulate our model for 200 quarters (i.e. 600 months). We repeat this exercise 10,000 times and report the average of the macroeconomic volatilities (HP filtered simulated data with smoothing parameter 100,000) in Table 4.

	U. Rate	Match. Rate	Sep. Rate	Product.
Standard Calibration				
Standard deviation	0.19	0.13	0.07	0.02
Relative to productivity	8.8	6.0	3.5	1
Quarterly autocorrelation	0.90	0.88	0.88	0.88

Table 4: Volatilities generated by the Incentive Model of Matching

The differences between our model and the conventional matching model are striking. Our model can generate the high macroeconomic volatilities found in the data. Our results are all the more remarkable, as we do not neither have to resort to Hall's (2005) real wage rigidity assumption nor to Hagedorn and Manovskii's (2008) small surplus calibration.

Specifically, the more rigid the wage in the conventional matching model (Hall, 2005), the greater the share of productivity variations that is captured by the firm and thus the greater the volatility of vacancies. However, there is evidence against the rigid-wage hypothesis both from the microeconomic and the macro perspective. Haefke et al. (2008) infer that wages for newly created jobs (i.e., those modeled in the matching model) are completely flexible on a microeconomic level. Hornstein et al. (2005) show that wages are roughly as volatile as the labor productivity on a macroeconomic level. By contrast, our model generates high

labor market volatilities, even though it replicates the stylized fact that wages are as volatile as productivity.

Hagedorn and Manovskii (2008) choose a small-surplus calibration to resolve the volatility puzzle of the matching model. Under this calibration, aggregate profits are only a very small share of the overall production in the steady state, so that a positive productivity shock sharply increases the relative profits. This gives a large incentive to firms to post more vacancies (due to the free entry condition). Consequently all labor market variables become volatile. This type of calibration has several shortcomings. Besides the unrealistically low profit share, the utility value of unemployment is extremely high and workers' bargaining power is very low in the calibration. Therefore workers are almost indifferent between working and not working. We do not need to rely on any of these mechanisms in our calibration. As noted, we assume that worker's bargaining power is 50 percent. The labor income divided by overall production is roughly 80 percent in our model. Furthermore, the average worker's disutility of labor and unemployment benefits make up only 80 percent of the current wage. As a consequence, the average worker is not indifferent between unemployment and employment.

7.2 Correlations

Our model features several additional advantages compared to the conventional matching framework. Krause and Lubik (2007) show that the matching framework with endogenous job destruction and flexible wages cannot generate a strong negative correlation between the job finding rate and the unemployment. In all of our model simulations, the correlation between these two variables is very strongly negative, in magnitude between -0.95 and -0.99, i.e., slightly higher than in the US data (-0.95, see Shimer, 2005).

Further, Krause and Lubik (2007) show that the matching model with endogenous job destruction and flexible wages cannot account for the negative correlation between job destruction and job creation. In our model, the correlation between these two variables is always negative and close to -1.²⁴

²⁴The job finding rate and the job destruction rate are both driven by the same underlying shock, resulting in this strong negative correlation. We could get a lower correlation if we introduced another shock to drive a wedge between the shocks underlying job destruction and those underlying job creation. However, for simplicity we do not choose this option.

8 Conclusion

This paper has presented a choice-theoretic theory of labor market matching in the presence of frictions, heterogeneous jobs and heterogeneous workers. This theory does not rely on a matching function. We have presented simple analytical models that derive labor market matches and separations from the optimizing behavior of workers and firms. Since the matching function is meant to encapsulate frictions and heterogeneities, we have examined whether it can replicate the optimizing behavior above. Our analysis indicates that the matching function is vulnerable to the Lucas critique, since it is not stable with respect to changes in policy and macroeconomic variables. Thus its use for policy analysis and prediction becomes problematic.

To keep our formal analysis as simple as possible, we have made some radically simplifying assumptions, such as those concerning wage determination, the depiction of heterogeneities in terms of only two additive shocks ε and e , and the depiction of adjustment costs in terms of only two additive costs h and f . Whereas these simplifying assumptions naturally affect the quantitative predictions of our model, they are not essential to basic idea that motivates this paper: namely, that the matching and separation probabilities can be understood in terms of job offer, job acceptance, firing, and quit probabilities, which may be derived from the optimizing decisions of firms and workers. These optimizing decisions - in the presence of heterogeneous workers and jobs, as well as costs of adjustment - explain why some job-seeking workers remain unemployed and some vacant jobs remain unfilled.

Needless to say, the incentive models presented above are merely a first step towards a choice-theoretic understanding of the matching process. Much research remains to be done. Although relaxing our simplifying assumptions regarding wage determination, heterogeneities and adjustment costs will not affect the basic idea above, it will help us refine the quantitative predictive properties of the incentive model.

Nevertheless, we have shown that even on the basis of our radically simplifying assumptions, our calibrated incentive model can account for various important empirical regularities that have eluded the conventional matching models. In particular, our model comes close to generating the empirically observed volatilities of the unemployment rate, the job finding rate and the separation rate. Furthermore, our model can also account for the observed strong negative correlations between the job finding rate and the unemployment rate, and between job creation and job destruction.

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A Appendix

A.1 Vacancies

Suppose that $V^* > U$, where V is aggregate vacancies and U is the unemployment level. The number of vacancies posted by firm φ is V_φ , and the number of vacancies posted by all other firms is $V_{-\varphi}$, so that $V = V_\varphi + V_{-\varphi}$. The probability that a vacancy is filled is $\rho = \frac{U}{V_\varphi + V_{-\varphi}}$. The number of job applicants at firm φ is

$$A_\varphi = V_\varphi \rho = \frac{UV_\varphi}{V_\varphi + V_{-\varphi}}. \quad (46)$$

The firm's expected profit is

$$E(\pi_\varphi) = \mu(a - w - h - E(\varepsilon)) \frac{UV_\varphi}{V_\varphi + V_{-\varphi}} - \kappa V_\varphi \quad (47)$$

where κ is the vacancy posting cost. Differentiating this profit function, we find the profit-maximizing number of vacancies:

$$V_\varphi^* = -\frac{V_{-\varphi}}{2} + \frac{1}{2\kappa} \left((\kappa V_{-\varphi})^2 - 4\kappa (\kappa V_{-\varphi}^2 - \mu(a - w - h - E(\varepsilon)) UV_{-\varphi}) \right)^{\frac{1}{2}} \quad (48)$$

There are $F + 1$ identical firms. Thus, in equilibrium, $V_{-\varphi} = FV_\varphi^*$, so that

$$V_\varphi^* = \frac{\mu(a - w - h - E(\varepsilon)) UF}{\kappa \left(1 + \frac{1}{2}F + F^2 \right)} \quad (49)$$

Observe that V_φ^* rises when (i) expected profit per worker $\mu(a - w - h - E(\varepsilon))$ rises, (ii) the number of unemployed U rises (a strategic complementarity, so that the "vacancy supply curve" is an upward-sloping line), (iii) the vacancy posting cost κ falls, and (iv) the number of firms F falls.

Analogously to the conventional matching function, we can assume that there is free entry of firms, so that the number of firms is such that expected profit is driven to zero:

$$E(\pi_\varphi) = \mu(a - w - h - E(\varepsilon)) \frac{UV_\varphi^*}{(1 + F)V_\varphi^*} - \kappa V_\varphi^* \quad (50)$$

and thus the equilibrium number of firms is

$$F^* = \mu(a - w - h - E(\varepsilon)) \frac{U}{\kappa V_\varphi^*} - 1 \quad (51)$$

The model above is compatible with unemployment, even though all unemployed workers find a vacancy. The reason is that not all vacancies lead to employment, since the match probability $\mu < 1$.

Finally, consider the case where $V < U$, where all job searchers are allocated randomly to vacancies. Then every vacancy is filled. Thus the firm's expected profit is

$$E(\pi_\varphi) = \mu(a - w - h - E(\varepsilon)) V_\varphi - \kappa V_\varphi \quad (52)$$

In this case, there is no interior solution to the profit-maximizing number of vacancies. If $\mu(a - w - h - E(\varepsilon)) > \kappa$, more vacancies will be created until $V > U$ (the case above). If $\mu(a - w - h - E(\varepsilon)) < \kappa$, then no vacancies are created, since the vacancy posting cost is too high.

A.2 Robustness

Table 5 provides a robustness analysis of the labor market volatilities implied by our model for values of the firing cost $f = 0.08$ and $f = 0.20$.

	U. Rate	Match. Rate	Sep. Rate	Product.
Volatilities for $f=0.08$				
Standard deviation	0.23	0.14	0.09	0.02
Relative to productivity	10.7	6.6	4.4	1
Quarterly autocorrelation	0.90	0.88	0.88	0.88
Volatilities for $f=0.2$				
Standard deviation	0.11	0.09	0.03	0.02
Relative to productivity	5.2	4.5	1.5	1
Quarterly autocorrelation	0.91	0.88	0.88	0.88

Table 5: Robustness Analysis of the Labor Market Volatilities Implied by our Model for Values of the Firing Cost $f = 0.08$ and $f = 0.20$.