

Non-nested Information Sets and the Term Structure of Interest Rates

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Relax the assumption that all traders have access to the same information

- ▶ Disagreement: It can be rational for individuals to believe that average expectations of future short rates are inaccurate
- ▶ Trade driven partly by speculative behavior in the sense of Harrison and Kreps (1978)
- ▶ No trader knows more (on average) than other traders

Provide a new explanation of predictable excess returns and reinterpret the statistical evidence about time varying price of risk

Non-nested Information Sets and the Term Structure of Interest Rates

A necessary condition for non-nested information sets is that bond yields are not Markov.

- ▶ Statistical evidence: Cochrane and Piazzesi (2005, 2008), Duffee (2008), Ludvigson and Ng (forthcoming).
- ▶ Theoretical argument: Grossman and Stiglitz (1983) impossibility of informationally efficient markets

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Non-nested information implies that expectations will be dispersed

- ▶ Survey of Professional Forecasters (Swanson JMCB 2006):
10th - 90th percentile spread is 80-220 basis points for 4 quarters ahead forecasts of 3 Month Treasury Bill yield (1992-2004 sample)

Information not known by others and therefore not yet reflected in prices is also more profitable, so traders *should* seek to obtain private information

Non-nested Information Sets and the Term Structure of Interest Rates

The punch-lines:

- ▶ Individual traders can predict excess returns when information sets are non-nested even in a model with constant price of risk
- ▶ Show that a dynamic model with non-nested information sets can be estimated with likelihood-based methods
- ▶ If the world is characterized by the model estimated in this paper, popular affine three factor no-arbitrage models will find overwhelming but misleading evidence in favor of a time varying price of risk

Excess returns and the expectations hypothesis

Expectations hypothesis (*w.r.t.* Ω_t) $\implies E[\gamma_t^n | \Omega_t] = 0$ or equivalently that $f_t^n = E[r_{t+n} | \Omega_t]$

$$y_t^n \equiv c^n + n^{-1} \sum_{s=0}^{n-1} E[r_{t+s} | \Omega_t] + \gamma_t^n$$
$$f_t^n \equiv \tilde{b}_t^n - \tilde{b}_t^{n+1}$$
$$= E[r_{t+n} | \Omega_t] + \gamma_t^n - \gamma_t^{n+1}$$

Here we define predictable excess returns as the failure of the expectations hypothesis to hold. The challenge has been to give economic meaning to γ_t^n

- ▶ Backus, Gregory and Zin (JME 1989) and Wachter (JFE 2006)

The inner-product space L^2

Definition

The inner product space L^2 is the collection \mathcal{C} of all random variables X with finite variance

$$EX^2 < \infty$$

and with inner-product

$$\langle X, Y \rangle \equiv E(XY) : X, Y \in L^2$$

In the model presented below, all bond yields, the factors that drive them and the signals that traders observe will be elements in L^2 .

Orthogonal Projections in L^2

Definition

Let Ω be a subspace of L^2 . An orthogonal projection of X on Ω , denoted $\mathcal{P}_\Omega X$, is the unique element in Ω satisfying

$$\langle X - \mathcal{P}_\Omega X, \omega \rangle = 0 : \omega \in \Omega$$

Orthogonal projections in L^2 minimizes variance of errors

4 useful properties of orthogonal projections

1. The projection $\mathcal{P}_\Omega X$ coincides with the expectation $E[X | \Omega]$ in linear models with Gaussian shocks.
2. Each $X \in L^2$ has a unique representation as a sum of an element in Ω and an element of Ω^\perp , i.e.

$$X = \mathcal{P}_\Omega X + (I - \mathcal{P}_\Omega) X$$

where Ω^\perp is the orthogonal complement of Ω in L^2

3. $X \in \Omega^\perp$ if and only if $\mathcal{P}_\Omega X = 0$.
4. $\Omega_1 \subseteq \Omega_2$ if and only if $\mathcal{P}_{\Omega_1} X = \mathcal{P}_{\Omega_1} \mathcal{P}_{\Omega_2} X$ for all $X \in L^2$.

Bond prices, forward rates and expectations

From an log-linear approximation to a wealth maximizer's portfolio choice we can derive the bond pricing equation

$$\tilde{b}_t^n = -r_t + \int E \left[\tilde{b}_{t+1}^{n-1} \mid \Omega_t(j) \right] dj - \eta_t^n$$

which can be used to find the forward rate f_t^n as a function of average higher order expectations of r_{t+n}

$$\begin{aligned} f_t^n &= \int \mathcal{P}_{t,j} \int \mathcal{P}_{t+1,j'} \dots \int \mathcal{P}_{t+n-1,j''} r_{t+n} + (\eta_t^n - \eta_t^{n+1}) \\ &\equiv \prod_{s=0}^{n-1} \int \mathcal{P}_{t+s,j} r_{t+n} + (\eta_t^n - \eta_t^{n+1}) \end{aligned}$$

Common information set Ω_t and the expectations hypothesis

Sequence $\{\Omega_t\}_{t=0}^{\infty}$ is a filtration: $\Omega_0 \subseteq \Omega_1 \subseteq \Omega_2 \dots \subseteq \Omega_t \dots$
Replace expectations with projections \mathcal{P}

$$\begin{aligned} f_t^2 &= E[E[r_{t+2} | \Omega_{t+1}] | \Omega_t] \\ &= \mathcal{P}_{\Omega_t} \mathcal{P}_{\Omega_{t+1}} r_{t+2} \end{aligned}$$

Property (5) of projections \implies

$$\begin{aligned} f_t^2 &= \mathcal{P}_{\Omega_t} r_{t+2} \\ &= E[r_{t+2} | \Omega_t] \end{aligned}$$

Traders do not expect to revise their expectations in a particular direction, or $\{E[r_{t+s} | \Omega_{t+s}]\}_{s=0}^n$ is a martingale.

Forward Rates and Traders' Expectations

Forward rates f_t^n coincide with individual trader expectation of r_{t+n} generally so that

$$\mathcal{P}_{t,j}r_{t+n} = f_t^n \quad \forall j, t, n$$

if and only if $\Omega_t(j) = \Omega_t(i) \quad \forall t, i, j$ since ex ante symmetry of traders implies that

$$\mathcal{P}_{t,j}r_{t+n} = \mathcal{P}_{t,i}r_{t+n} \quad \forall j, i, t, n$$

or that the forward rate is the best prediction of trader j only when it is also the best prediction for all others traders, i.e. when information sets are nested.

- ▶ Uniqueness of orthogonal projections imply that individual traders can predict excess returns since $\mathcal{P}_{t,j}(f_t^n - r_{t+n}) \neq 0$

Predicting average expectations errors

Trader j can systematically predict average projection errors of the short rate

$$\underbrace{\mathcal{P}_{t,j} \left(r_{t+n} - \int \mathcal{P}_{t+s,j'} r_{t+n} dj' \right)}_{\text{Average prediction error}} \neq 0 \quad (1)$$

since (1) can be rearranged to

$$\mathcal{P}_{t,j} r_{t+n} \neq \mathcal{P}_{t,j} \mathcal{P}_{t+s,i} r_{t+n}$$

using that $\mathcal{P}_{t,j} \int \mathcal{P}_{t+s,j'} r_{t+n} dj' = \mathcal{P}_{t,j} \mathcal{P}_{t+s,i} r_{t+n}$.

- ▶ By property (4) above we know that $\mathcal{P}_{t,j} r_{t+n} = \mathcal{P}_{t,j} \mathcal{P}_{t+s,i} r_{t+n}$ if and only if $\Omega_t(j) \subseteq \Omega_{t+s}(i)$.

Forward Rates and Expectations

The forward rate f_t^n can be decomposed into the projection of r_{t+n} onto public information set Ω_t^p , terms that are orthogonal to Ω_t^p and exogenous supply shocks η_t^n and η_t^{n+1} .

$$\begin{aligned} f_t^n &= \mathcal{P}_t^p r_{t+n} + \int \mathcal{P}_{t,j}^{\perp p} r_{t+n} \\ &+ \sum_{m=1}^n \prod_{s=0}^{n-m} \int \mathcal{P}_{t+s,j}^{\perp p} \left(r_{t+n} - \int \mathcal{P}_{t+m-1,j} r_{t+n} \right) \\ &+ (\eta_t^n - \eta_t^{n+1}) \end{aligned}$$

That is, non-nested information introduces new dynamics to bond yields, however, these are orthogonal to public information (e.g. bond prices).

An Empirical Model

Short rate follows

$$r_t = x_t^1 + x_t^2 + \phi r_{t-1}$$

where

$$\begin{bmatrix} x_t^1 \\ x_t^2 \end{bmatrix} = \begin{bmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{bmatrix} \mathbf{x}_{t-1} + \Sigma \varepsilon_t$$

Private signal of trader j

$$s_t(j) = x_t^1 + \zeta(j)$$

In period t trader j observes the vector $S_t(j)$

$$S_t(j) = [s_t(j) \quad r_t \quad \mathbf{y}'_t]'$$

Model solved using method in Nimark (2007)

Estimation Method and Data

- ▶ Use Fed Funds Rate, 3, 12, 24, 36, 48, 60 month bond yields from CRSP (Fama-Bliss)
- ▶ Monthly data, 1964:1-2007:12 (528 obs)
- ▶ Time series demeaned
- ▶ Simulate posterior distribution using Adaptive Metropolis Algorithm (Haario et al 2001)
- ▶ Uniform (truncated) priors

Estimating the model

Parameter Estimates 1964:1-2007:12

	Mode	2.5%-97.5%		Mode	2.5%-97.5%
Short rate process			Bond supply shocks		
ρ_1	0.998	0.997 - 0.999	η_3	0.70	0.65 - 0.74
ρ_2	0.38	0.33 - 0.43	η_{12}	0.37	0.35 - 0.40
ϕ	0.84	0.82 - 0.85	η_{24}	0.16	0.16 - 0.18
σ_1	0.08	0.07 - 0.09	η_{36}	0.0002	0.0001 - 0.0021
σ_2	0.52	0.49 - 0.56	η_{48}	0.15	0.14 - 0.16
σ_{12}	-0.10	(-0.14) - (-0.05)	η_{50}	0.24	0.22 - 0.25
Noise in private signal					
σ_ζ	0.00001	0.00005 - 0.005			

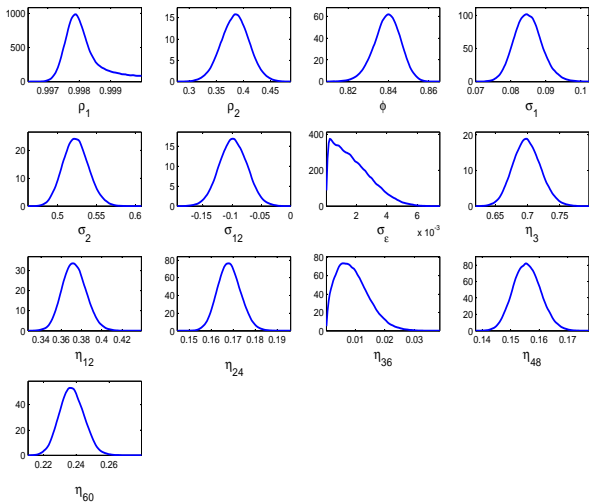


Figure: Estimated posterior densities of structural parameters

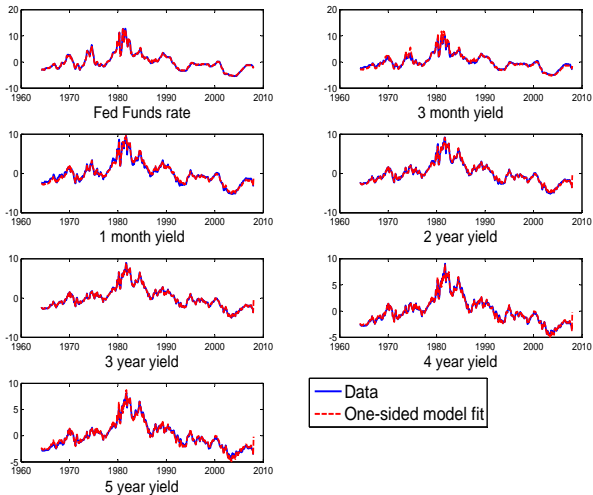


Figure: Data (blue solid) and one-sided model fit (red dashed).

The model and statistical measures of time varying price of risk

The three factors in the vector f_t follow

$$\begin{aligned}f_t &= \phi f_{t-1} + \Sigma e_t \\r_t &= \delta' f_{t-1}\end{aligned}$$

Log bond price deviations are then given by

$$\tilde{b}_t^n = B_n' f_t$$

where

$$B_n' = B_{n-1}' (\phi - \Sigma \lambda) - \delta'$$

Estimate 3 factor no-arbitrage model on artificial data from posterior of non-nested info model and compare log-likelihoods of $\lambda = 0$ and λ unrestricted

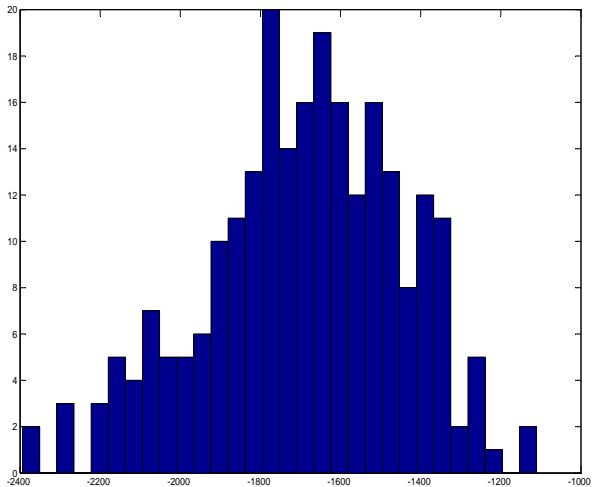


Figure: Difference of log likelihood of restricted and unrestricted model

The Estimated Systematic Component of Risk Premia

Duffee (2008) estimates the 5 factor model

$$\begin{aligned}x_t^\dagger &= D^\dagger x_{t-1}^\dagger + \Sigma^\dagger \epsilon_t \\y_t &= A + B^\dagger x_t^\dagger + \eta_t^\dagger\end{aligned}$$

- ▶ Can be rotated to compute the implied principal components.
- ▶ Duffee finds that while the first three principal components explain almost all of the unconditional variation in yields, the fifth principal component is important for explaining expected future short rates.
- ▶ IRF of shocks to 5th principal component will originate at zero

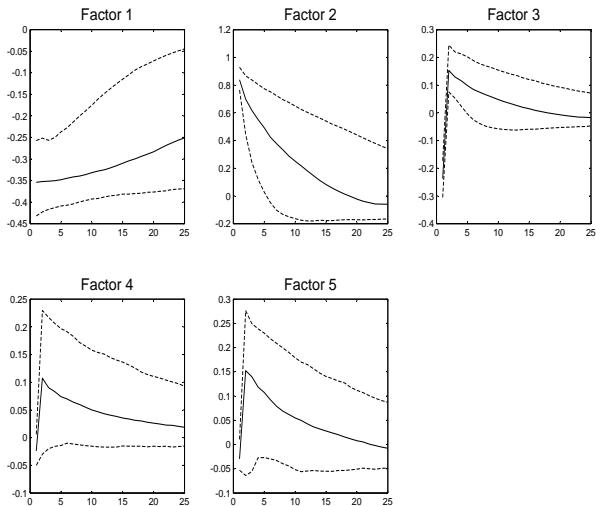


Figure: Impulse responses to orthogonal factor innovations.

Conclusions

- ▶ Non-nested information sets induces “speculative behavior” among rational traders and may explain predictable excess returns without relying on a time varying price of risk
 - ▶ How much information is public in the strong common knowledge sense of the word?
- ▶ We can take dynamic models with privately informed agents directly to the data.
 - ▶ More things to be done with estimated model: Quantify dispersion of expectations and gains from private information
- ▶ Model can account for evidence from statistical models of the term structure
 - ▶ Apparent time varying risk premia may be due to misspecification
 - ▶ Duffee’s (2008) “hidden factor” is a natural feature of a term structure model with imperfectly informed traders