# Bayesian Analysis of DSGE Models

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# A Prototypical DSGE Model

- Detailed discussion provided in Woodford (2003).
- Households: consume, competitively supply labor services, value transaction services from real money balances,
- Final Goods Producers: aggregate a continuum of intermediate goods.
- Intermediate Goods Producers: use capital services and labor, AR(1) technology. Monopolistic competition and Calvo-style nominal rigidities.
- Monetary Policy: represented through interest rate feedback rule.

# A Prototypical DSGE Model

- Final good production:

$$Y_t = \left( \int_0^1 Y_t(j)^{1-\nu} dj \right)^{\frac{1}{1-\nu}}. \tag{1}$$

- Demand for intermediate inputs:

$$Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-1/\nu} Y_t. \tag{2}$$

- Price of final good:

$$P_t = \left( \int_0^1 P_t(j)^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}. \tag{3}$$

# A Prototypical DSGE Model

- Intermediate goods are produced by monopolistically competitive firms with the following technology:

$$Y_t(j) = A_t N_t(j), \tag{4}$$

- $A_t$ is an exogenous productivity process that is common to all firms and $N_t(j)$ is the labor input of firm $j$.
- Labor is hired in a perfectly competitive factor market at the real wage $W_t$.
- Firms face nominal rigidities in terms of quadratic price adjustment costs

$$AC_t(j) = \frac{\varphi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - \pi\right)^2 Y_t(j), \tag{5}$$

# A Prototypical DSGE Model

- Firm $j$ chooses its labor input $N_t(j)$ and the price $P_t(j)$ to maximize the present value of future profits

$$\mathbb{E}_t \left[ \sum_{s=0}^{\infty} \beta^s Q_{t+s} \left( \frac{P_{t+s}(j)}{P_{t+s}} Y_{t+s}(j) - W_{t+s} N_{t+s}(j) - AC_{t+s}(j) \right) \right]. \tag{6}$$

- The household maximizes

$$\mathbb{E}_t \left[ \sum_{s=0}^{\infty} \beta^s \left( \frac{(C_{t+s}/A_{t+s})^{1-\tau} - 1}{1-\tau} + \chi_M \ln \left( \frac{M_{t+s}}{P_{t+s}} \right) - \chi_H H_{t+s} \right) \right], \tag{7}$$

- subject to the budget constraint

$$P_t C_t + B_t + M_t - M_{t-1} + T_t = P_t W_t H_t + R_{t-1} B_{t-1} + P_t D_t \tag{8}$$

# A Prototypical DSGE Model

- Monetary policy is described by an interest rate feedback rule of the form

$$R_t = R_t^{*\,1-\rho_R} R_{t-1}^{\rho_R} e^{\epsilon_{R,t}}, \tag{9}$$

- Output gap rule:

$$R_t^* = r\pi^* \left(\frac{\pi_t}{\pi^*}\right)^{\psi_1} \left(\frac{Y_t}{Y_t^*}\right)^{\psi_2} \quad \text{(output gap rule specification } \mathcal{M}_1\text{)} \tag{10}$$

- Output growth rule:

$$R_t^* = r\pi^* \left(\frac{\pi_t}{\pi^*}\right)^{\psi_1} \left(\frac{Y_t}{\gamma Y_{t-1}}\right)^{\psi_2} \quad \text{(output growth rule specification } \mathcal{M}_2\text{).} \tag{11}$$

- Government budget constraint

$$P_t G_t + R_{t-1} B_{t-1} = T_t + B_t + M_t - M_{t-1}. \tag{12}$$

# A Prototypical DSGE Model

- Exogenous processes...

- Technology evolves according to

$$\ln A_t = \ln \gamma + \ln A_{t-1} + \ln z_t, \quad \text{where} \quad \ln z_t = \rho_z \ln z_{t-1} + \epsilon_{z,t}. \tag{13}$$

- Government spending: define $g_t = 1/(1 - \zeta_t)$. We assume

$$\ln g_t = (1 - \rho_g) \ln g^* + \rho_g \ln g_{t-1} + \epsilon_{g,t}. \tag{14}$$

- Monetary policy shock is serially uncorrelated.

- Define: $\epsilon_t = [\epsilon_{R,t}, \epsilon_{g,t}, \epsilon_{z,t}]'$

# A Prototypical DSGE Model

- Non-stationary technology process $A_t$ induces a stochastic trend in output and consumption.

- Express the model in terms of detrended variables $c_t = C_t/A_t$ and $y_t = Y_t/A_t$.

- The steady state inflation $\pi$ equals the target rate $\pi^*$ and

$$r = \frac{\gamma}{\beta}, \quad R = r\pi^*, \quad c = (1-\nu)^{1/\tau}, \quad \text{and} \quad y = g(1-\nu)^{1/\tau}. \tag{15}$$

  Let $\hat{x}_t = \ln(x_t/x)$ denote the percentage deviation of $x_t$ from its steady state $x$.

# A Prototypical DSGE Model

- The DSGE model can be expressed as

$$
\begin{aligned}
1 &= I\!E_t \left[ e^{-\tau \hat{c}_{t+1} + \tau \hat{c}_t + \hat{R}_t - \hat{z}_{t+1} - \hat{\pi}_{t+1}} \right] && (16) \\
\frac{1-\nu}{\nu \varphi \pi^2} \left( e^{\tau \hat{c}_t} - 1 \right) &= \left( e^{\hat{\pi}_t} - 1 \right) \left[ \left( 1 - \frac{1}{2\nu} \right) e^{\hat{\pi}_t} + \frac{1}{2\nu} \right] && (17) \\
&\quad - \beta\, I\!E_t \left[ \left( e^{\hat{\pi}_{t+1}} - 1 \right) e^{-\tau \hat{c}_{t+1} + \tau \hat{c}_t + \hat{y}_{t+1} - \hat{y}_t + \hat{\pi}_{t+1}} \right] \\
e^{\hat{c}_t - \hat{y}_t} &= e^{-\hat{g}_t} - \frac{\varphi \pi^2 g}{2} \left( e^{\hat{\pi}_t} - 1 \right)^2 && (18) \\
\hat{R}_t &= \rho_R \hat{R}_{t-1} + (1-\rho_R)\psi_1 \hat{\pi}_t + (1-\rho_R)\psi_2 \left( \hat{y}_t - \hat{g}_t \right) + \epsilon_{R,t} && (19) \\
\hat{g}_t &= \rho_g \hat{g}_{t-1} + \epsilon_{g,t} && (20) \\
\hat{z}_t &= \rho_z \hat{z}_{t-1} + \epsilon_{z,t} && (21)
\end{aligned}
$$

- For output growth rule specification:

$$
\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\psi_1 \hat{\pi}_t + (1-\rho_R)\psi_2 \left( \Delta \hat{y}_t + \hat{z}_t \right) + \epsilon_{R,t}. \qquad (22)
$$

# A Prototypical DSGE Model

- Define

$$s_t = [\hat{y}_t, \hat{c}_t, \hat{\pi}_t, \hat{R}_t, \epsilon_{R,t}, \hat{g}_t, \hat{z}_t]'.$$

- The solution of the rational expectations system takes the form

$$s_t = \Phi(s_{t-1}, \epsilon_t; \theta). \tag{23}$$

- Measurement equations

$$\begin{aligned} YGR_t &= \gamma^Q + \hat{y}_t - \hat{y}_{t-1} + \hat{z}_t \qquad (24) \\ INFL_t &= \pi^{(A)} + 4 \times \hat{\pi}_t \\ INT_t &= \pi^{(A)} + r^{(A)} + 4 \times \gamma^{(Q)} + 4 \times \hat{R}_t. \end{aligned}$$

# A Prototypical DSGE Model

- The parameters $\gamma^Q$, $\pi^{(A)}$, and $r^{(A)}$ are related to the steady states of the model economy as follows

$$\gamma = 1 + \frac{\gamma^{(Q)}}{100}, \quad \beta = \frac{1}{1 + r^{(A)}/400}, \quad \pi = 1 + \frac{\pi^{(A)}}{400}.$$

- The structural parameters are collected in the vector $\theta$. Since in the first-order approximation the parameters $\nu$ and $\varphi$ are not separately identifiable, we express the model in terms of the slope of the Phillips curve $\kappa$. Let

$$\theta = [\tau, \kappa, \psi_1, \psi_2, \rho_R, \rho_g, \rho_z, r^{(A)}, \pi^{(A)}, \gamma^{(Q)}, \sigma_R, \sigma_g, \sigma_z]'.$$