Bayesian Methods for Macroeconometrics

Frank Schorfheide
Department of Economics, University of Pennsylvania

Model (I)

- New Keynesian IS-LM model: e.g., Gali and Gertler (1999), King (2000), Woodford (2000). Three shocks: technology, government spending / preference, monetary policy.

- Departures from standard model:

  – Monetary policy:

$$R_t = (R_t^*)^{1-\rho_R} R_{t-1}^{\rho_R} \exp\{\epsilon_{R,t}^*\}, \quad R_t^* = (r\pi_t^*)\left(\frac{\pi_t}{\pi_t^*}\right)^{\psi}. \tag{1}$$

  where

$$\ln \pi_t^*(s_t) = \begin{cases} \ln \pi_L^* & \text{if } s_t = 1 \\ \ln \pi_H^* & \text{if } s_t = 2 \end{cases}, \quad \mathcal{P} = \begin{bmatrix} \phi_1 & 1-\phi_2 \\ 1-\phi_1 & \phi_2 \end{bmatrix}. \tag{2}$$

  – Heteroskedasticity: variance of policy shock $\epsilon_{R,t}$ is regime dependent.

  – Agents have to learn regime s_t based on past observations.

Model (II)

- Departures from standard model (continued):
  - Technology: random walk with drift, serially correlated growth rates.
- Log-linear approximation of policy rule:

$$\ln \frac{R_t}{R} = (1-\rho_R)\ln \frac{\pi_t^*}{\pi} + (1-\rho_R)\psi[\ln \frac{\pi_t}{\pi} - \ln \frac{\pi_t^*}{\pi}] + \rho_R \ln \frac{R_{t-1}}{R} + \epsilon_{R,t}^*. \qquad (3)$$

  Thus,

$$\widetilde{R}_t = (1-\rho_R)\psi\widetilde{\pi}_t + \rho_R\widetilde{R}_{t-1} + \epsilon_{R,t}, \quad \epsilon_{R,t} = (1-\rho_R)(1-\psi)\widetilde{\pi}_t^* + \epsilon_{R,t}^*. \qquad (4)$$

- Note: time variation in ψ, e.g., Clarida, Gali, Gertler (2000), Lubik and Schorfheide (2002), is second order relative to the variation in π^* and omitted from analysis.

Model (III)

- A log-linear approximation of the market-clearing conditions and the first-order conditions of the household's and firms' problems leads to the following two equations:

$$\widetilde{y}_t = I\!E_t[\widetilde{y}_{t+1}] - \tau(\widetilde{R}_t - I\!E_t[\widetilde{\pi}_{t+1}]) - I\!E_t[\Delta\widetilde{g}_{t+1}] + \tau I\!E_t[z_{t+1}], \tag{5}$$

$$\widetilde{\pi}_t = \frac{e^{\gamma}}{r} I\!E_t[\widetilde{\pi}_{t+1}] + \kappa[\widetilde{y}_t - \widetilde{g}_t]. \tag{6}$$

Model: Approximate Solution

- Vector of relevant model variables

$$x_t = [\widetilde{y}_t, \widetilde{\pi}_t, \widetilde{R}_t, \ldots]',$$

- Vector of exogenous shocks (Note: $\epsilon_{R,t}$ is the composite policy shock)

$$\epsilon_t = [\epsilon_{g,t}, \epsilon_{z,t}, \epsilon_{R,t}(s_t)]'.$$

- Approximate solution of linear rational expectations model is of the form:

$$x_t = \Theta_1 x_{t-1} + \Theta_0 \epsilon_t + \Theta_x \sum_{j=1}^{\infty} \Theta_f^{j-1} \Theta_\epsilon I\!E_t[\epsilon_{t+j}]. \tag{7}$$

- Under 'learning':

$$x_t = \Theta_1 x_{t-1} + \Theta_0 \epsilon_t(s_t) + f_l\big(\epsilon_R^t(s_t)\big). \tag{8}$$

- Under 'full information' about state of policy in current period:

$$x_t = \Theta_1 x_{t-1} + \Theta_0 \epsilon_t(s_t) + f_f(s_t). \tag{9}$$

Econometrics: Posterior Inference

- Model will be fitted to output growth, inflation, interest rates. Measurement equation:

$$y_t = A_0 + A_1 x_t.$$

- Structural parameters: θ; switching probabilities: ϕ. Data Y^T, Markov States S^T.

- Posterior inference through Bayes Theorem:

$$p(\theta, \phi, S^T | Y^T) = \frac{p(Y^T | \theta, \phi, S^T) p(S^T | \phi) p(\phi, \theta)}{p(Y^T)}, \tag{10}$$

- We factorize the posterior as

$$p(\theta, \phi, S^T | Y^T) = \underbrace{p(\theta, \phi | Y^T)}_{\text{Metropolis-Hastings}} * \underbrace{p(S^T | \theta, \phi, Y^T)}_{\text{Kim's Smoother}} \tag{11}$$

Econometrics: Likelihood Function (I)

- Typically one uses Kalman filter for linear state-space model:

$$\underbrace{p(x_t|Y^t)}_{\text{Initialization}} \longrightarrow \underbrace{p(x_{t+1}|Y^t), \quad p(y_{t+1}|Y^t)}_{\text{Forecasting}} \longrightarrow \underbrace{p(x_{t+1}|Y^{t+1})}_{\text{Updating}} \tag{12}$$

- Presence of Markov-switching complicates computation of various conditionals.

- 'Learning' specification (depends on s_t only through $\epsilon_{R,t}$):

  – Composite policy shock can be recovered from observables conditional on θ, ϕ:

$$\epsilon_{R,t}(s_t) = \underbrace{\ln R_t - (1-\rho_R)\psi \ln \pi_t}_{\lambda' y_t} - \rho_R \ln R_{t-1} - (1-\rho_R)\ln r - (1-\rho_R)(1-\psi)\ln \pi^*.$$

  – Factorize

$$p(y_{t+1}|Y^t) = \underbrace{p(\lambda' y_{t+1}|Y^t)}_{\text{doesn't depend on } X^t} \quad * \quad \underbrace{p(y_{t+1}|\lambda' y_{t+1}, Y^t)}_{\text{doesn't depend on } s_{t+1}} \tag{13}$$

  and use standard Kalman filter formula for 2nd term.

Econometrics: Likelihood Function (II)

- 'Full-information' specification:
  - Markov state s_t enters the conditional distribution of x_t directly.
  - The distribution $x_t|Y^t$ is a Gaussian mixture with 2^t components.
  - To keep filter operable: collapse some of the mixture components at the end of each iteration (see Kim and Nelson (1999)).
  - We keep 2^k components, where $k = 3$.
  - Eliminate components with weight less than 10^{-5}.
  - Aggregate components with similar histories $s_{t+1}, \ldots, s_{t-k+2}$.
  - Accuracy of log likelihood at posterior mode $> 99.99\%$ based on $k = 3$ vs. $k = 20$.

Empirical Analysis: Data

- Quarterly post-war U.S. data from 1960:I to 1997:IV
  - *Output Growth* y_t: log differences of real per capita GDP (GDPQ), multiplied by 100 to convert into percent, not annualized.
  - *Inflation* π_t: annualized percentage change of CPI-U (PUNEW).
  - *Nominal Interest* R_t: average Federal Funds Rate (FYFF) in percent, demeaned.

Empirical Analysis: Priors

- 90% Confidence interval (CI) for ψ: 1 to 2.2. Prior distribution is truncated at the boundary of indeterminacy region.

- I identify regime $s_t = 1$ as 'low inflation' regime with π_L^* and impose that $\pi_H^* \geq \pi_L^*$. Prior CI for $\ln \pi_L *$ and $\ln \pi_H^*/\pi_L^*$: 1.3 to 4.5%.

- Standard deviation for $\epsilon_{R,t}^*$: 25 to 100 basis points (annualized).

- Prior for transition probabilities implies that duration lies between 6 and 50 quarters.

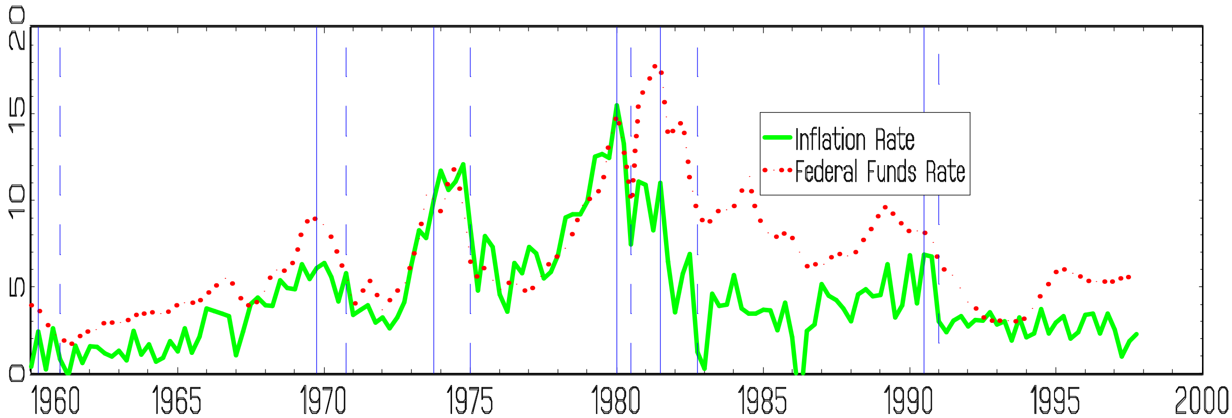
- Prior mean for annualized real interest rate is 2%.

Empirical Analysis: Selected Parameter Estimates

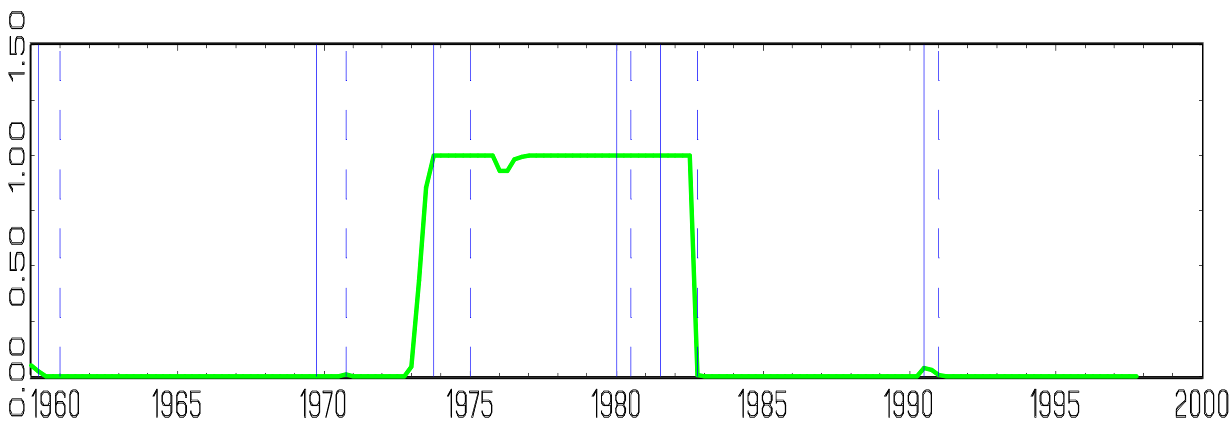
- Difference in target inflation: 4.8% under ‘learning’, 5.2% under ‘full information’.
- Probability of regime shift is less than 5%, duration of regime more than 5 years.
- High-inflation-target regime coincides with high volatility of monetary policy shock and more erratic behavior of Fed.
- In the absence of regime switching, the estimate of ψ is much closer to one. Inflation enters reaction function through $\psi \ln \pi_t/\pi_t^*$. High inflation in the 1970’s:
  - Interpretation 1: small ψ, large deviation from target.
  - Interpretation 2: large ψ, small deviation from target due to higher target.
  - Here I ignore possible time variation in ψ. Lubik and Schorfheide (2002) find evidence for inactive policy in the 1970’s even after adjusting for different targets.

	No Switching		Full Information		Learning	
	Mean	Conf. Interval	Mean	Conf Interval	Mean	Conf Interval
$\ln \pi_L^*$	4.36	[3.62, 5.07]	2.83	[2.19, 3.49]	2.63	[1.87, 3.38]
$\ln \pi_H^*/\pi_L^*$			5.21	[3.94, 6.53]	4.78	[3.21, 6.28]
$\ln r$	2.10	[1.41, 2.82]	2.21	[1.68, 2.73]	2.20	[1.59, 2.82]
ψ	1.14	[1.00, 1.30]	1.68	[1.41, 1.96]	1.77	[1.32, 2.18]
ρ_R	0.82	[0.78, 0.87]	0.75	[0.70, 0.79]	0.76	[0.71, 0.81]
$\sigma_{R,L}$	0.99	[0.89, 1.08]	0.68	[0.59, 0.77]	0.62	[0.48, 0.74]
$\sigma_{R,H}$			1.71	[1.39, 2.02]	1.57	[1.31, 1.82]
ϕ_1			0.97	[0.96, 0.99]	0.96	[0.93, 0.99]
ϕ_2			0.95	[0.93, 0.98]	0.95	[0.92, 0.99]

Inflation and Interest Rates



Prob (High Infl): Full-Information



Prob (High Infl): Learning