Robust Monetary Policy with Imperfect Knowledge

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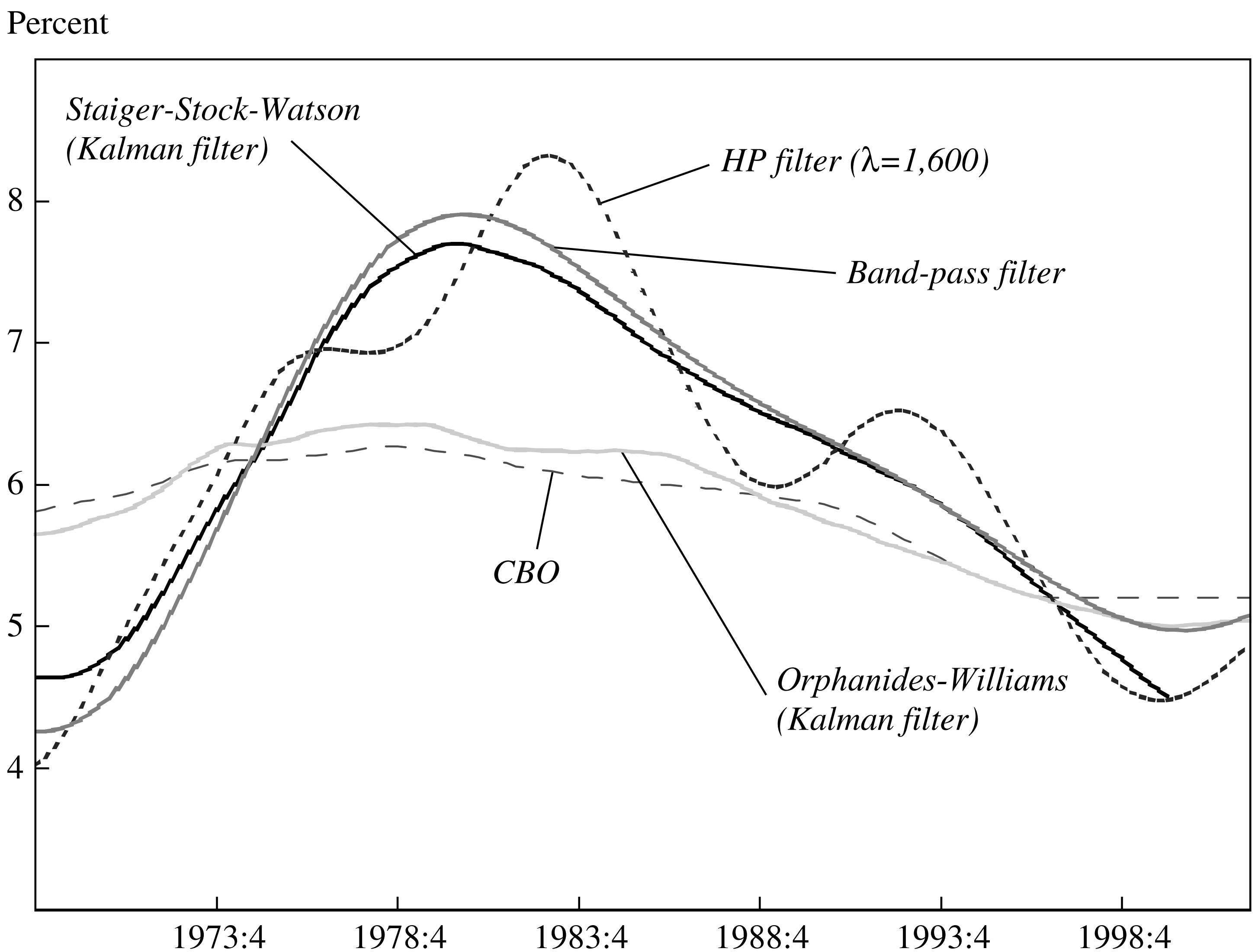
Imperfect Knowledge

- Basic measurement problems
  - What is "real output"
  - How do we measure "the" price index and inflation
- How does the economy work? (Model Structure)
- Limited knowledge of natural and equilibrium concepts
  - Natural rate of unemployment/potential output
  - Natural rate of interest
  - Equilibrium asset prices/exchange rates
- Expectations formation
  - "Rational" (model consistent)
  - Extrapolative (fixed coefficient adaptive)
  - Adaptive (with learning)

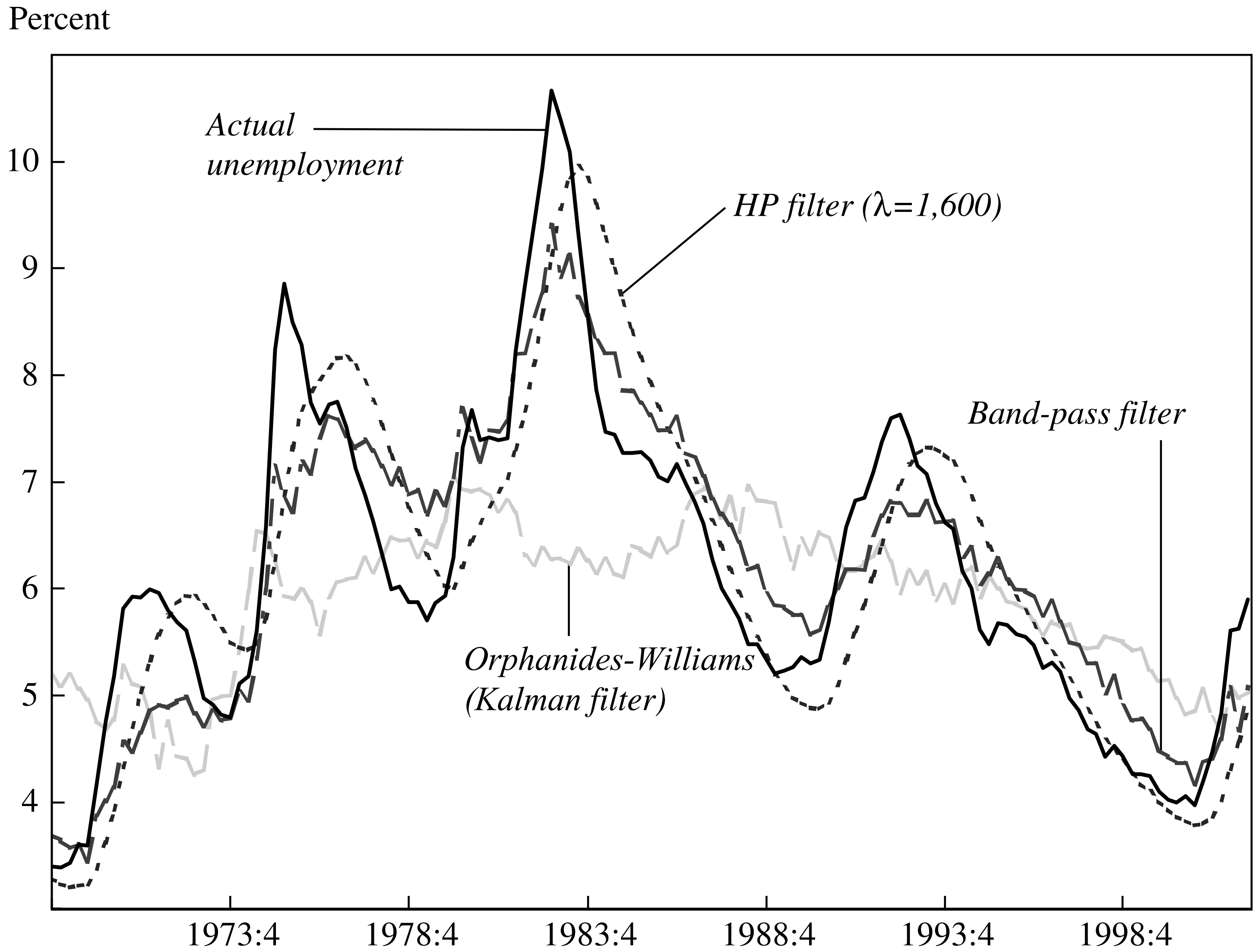
The Information Problem

Contemporary interpreters of the course of business have notoriously failed not only to predict the course of business but even to identify the current state of affairs. It is not abnormal for some to assert that we are in the early stages of deflation and others that we are entering into an inflation. (Milton Friedman, 1947)

Retrospective Estimates of the Natural Rate of Unemployment



Real-Time Estimates of the Natural Rate of Unemployment



The Nature of the Information Problem

Consider the Taylor Rule as an example:

Intended Policy:

$$i_t = r^* + \pi + \theta_\pi(\pi - \pi^*) + \theta_u(u - u^*)$$

Perceptions versus Reality:

$$\hat{u}^* = u^* + \eta_{u^*}$$
$$\hat{r}^* = r^* + \eta_{r^*}$$

Specification of rule in practice based on perceptions:

$$i_t = \hat{r}^* + \pi + \theta_\pi(\pi - \pi^*) + \theta_u(u - \hat{u}^*)$$

The Nature of the Information Problem

Resulting policy in practice:

$$i = r^* + \pi + \theta_\pi(\pi - \pi^*) + \theta_u(u - u^*) - \underbrace{((r^* - \hat{r}^*) - \theta_u(u^* - \hat{u}^*))}_{\text{misperceptions error}}$$

- Rule "prescription" sensitive to misperceptions.
- Historical descriptions of policy need to account for policymaker misperceptions.
- Informational limitations must be accounted for policy evaluation and design.
- Aggressive response to mismeasured natural rate "gaps" makes policy a source of noise.

Monetary Policy Design

Generalized Taylor Rule:

$$i = \theta_i i_{-1} + (1-\theta_i)(\hat{r}^* + \pi) + \theta_\pi(\pi - \pi^*) + \theta_u(u - \hat{u}^*) + \theta_{\Delta u}(u - u_{-1})$$

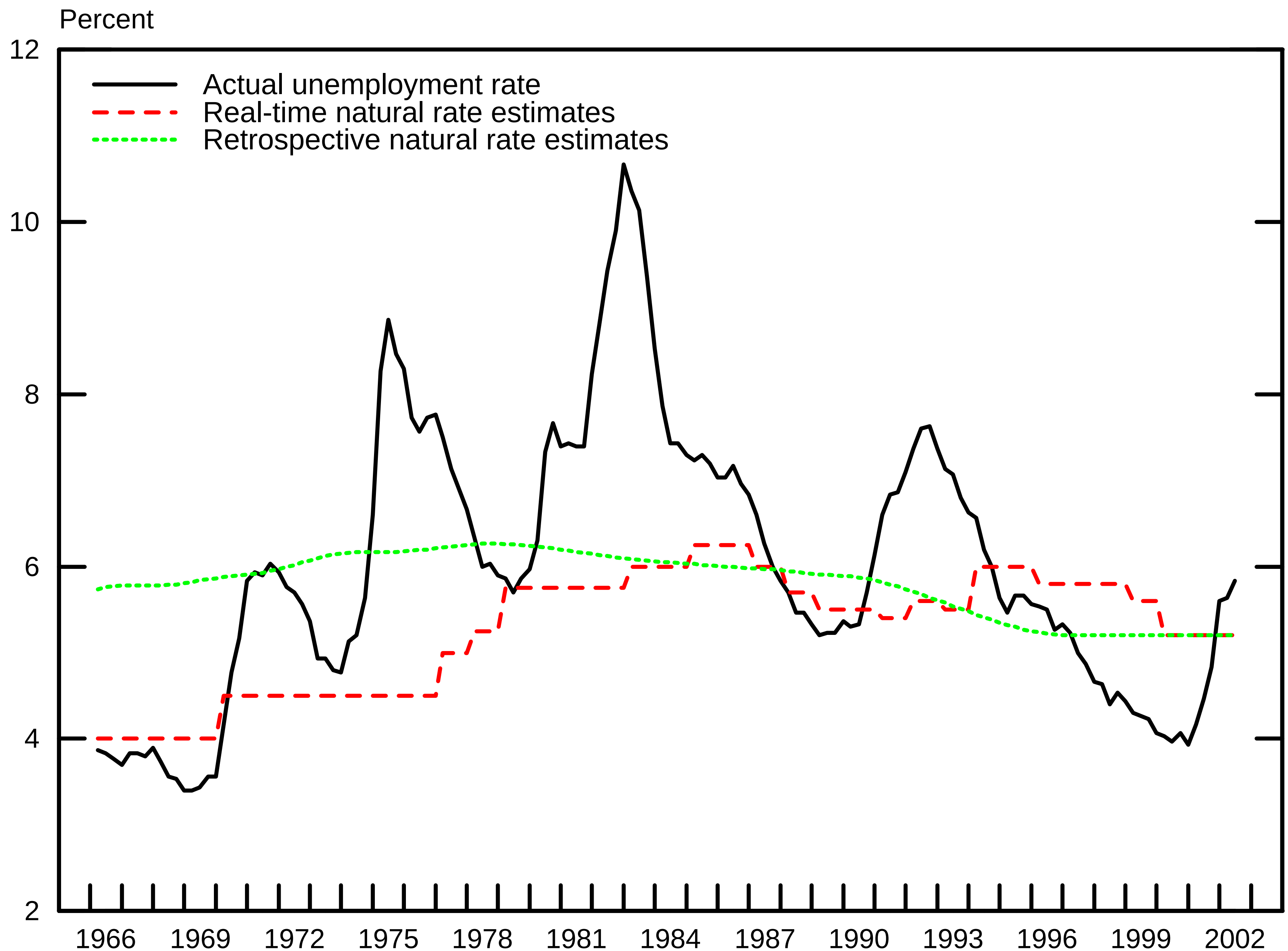
Misperceptions:

$$(1 - \theta_i)(r^* - \hat{r}^*) - \theta_u(u^* - \hat{u}^*)$$

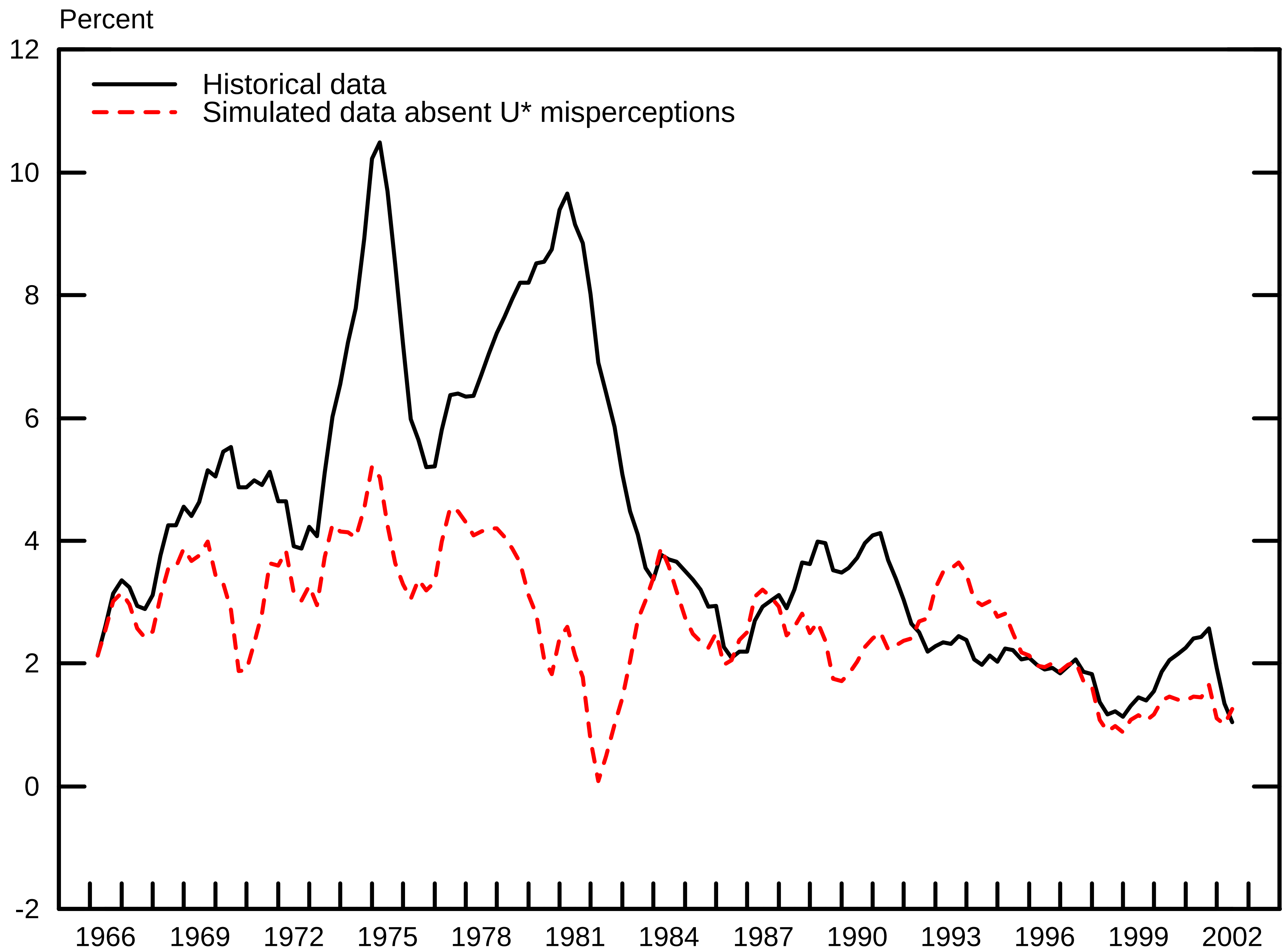
Difference Rule: $\theta_i \to 1, \quad \theta_u \to 0$

$$\Delta i = \theta_\pi(\pi - \pi^*) + \theta_{\Delta u}\Delta u$$

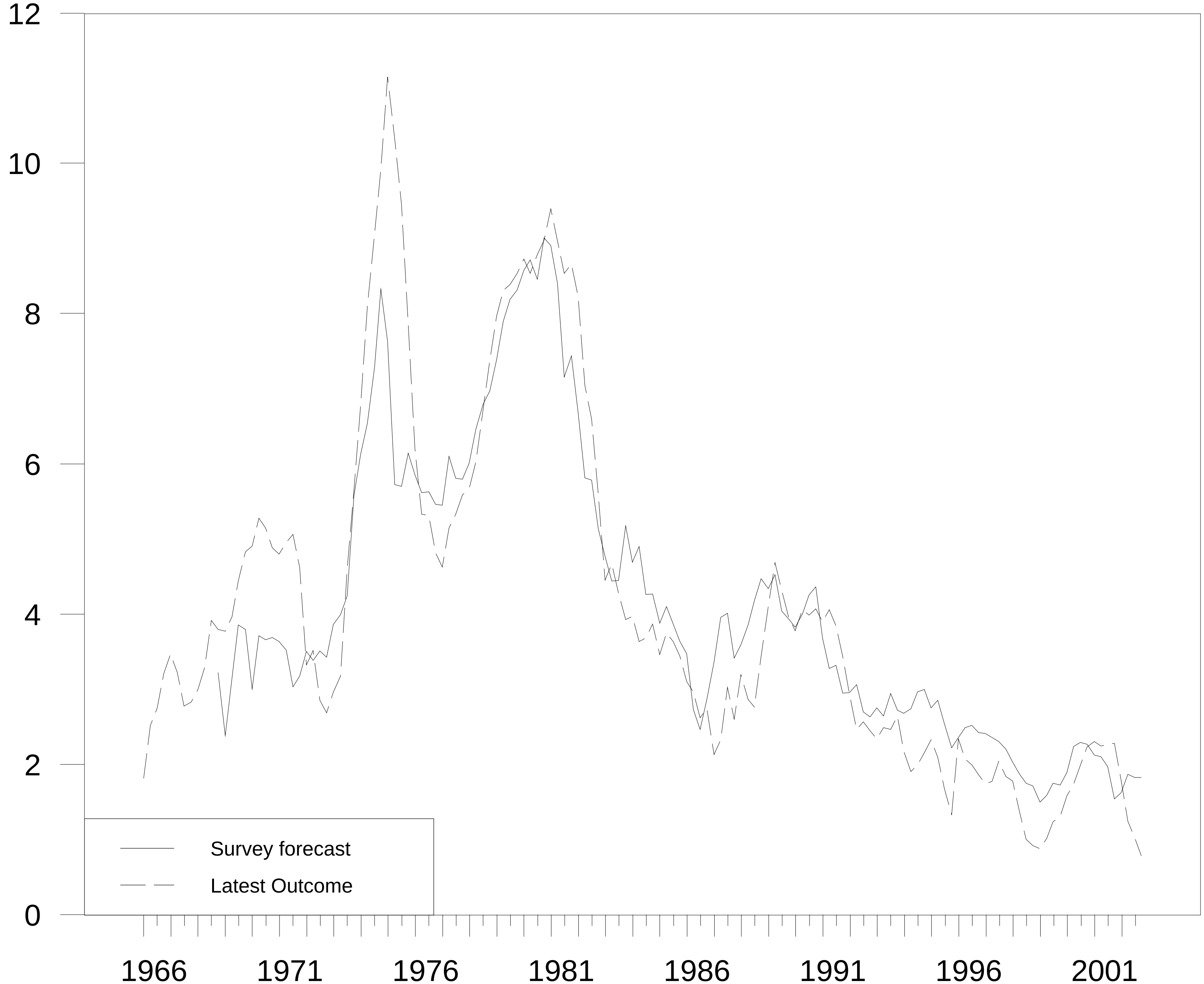
Unemployment and Historical Natural Rate Estimates



Inflation: History vs Counterfactual Experiment



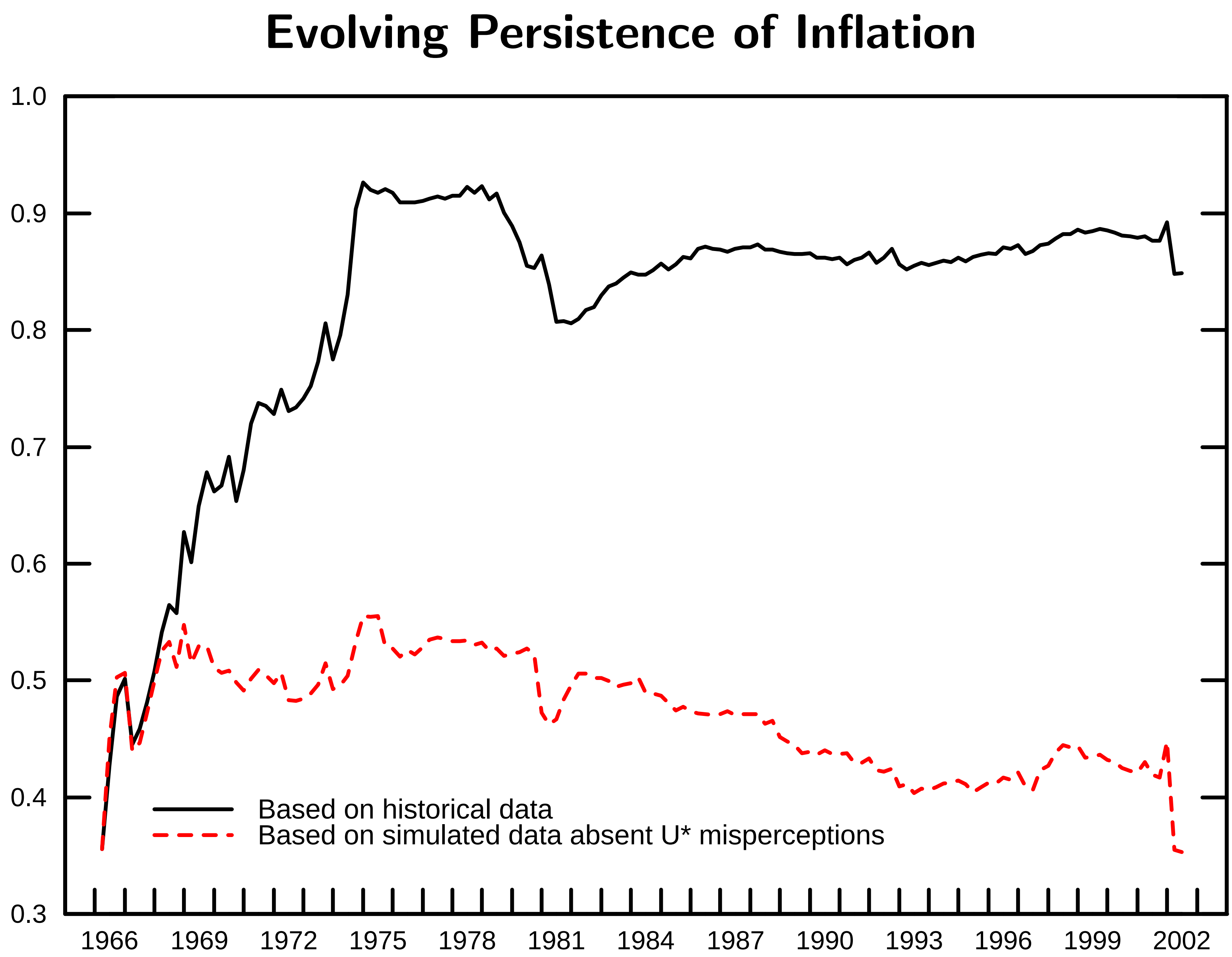
Inflation: latest outcome and year-ahead forecast



Inflation (output deflator) from $t-5$ to $t-1$, and forecast from $t-1$ to $t+3$ from the SPF survey at t.

Imperfections in Expectations Formation

- Recognize imperfections in expectations formation that yield relatively modest deviations from the "perfect knowledge" rational expectations benchmark.
- Formalize imperfect knowledge as a process of perpetual learning by private agents.
- Investigate the role of learning as a propagation mechanism, in terms of amplification and persistence.
- Examine interactions of learning with policy errors due to misperceptions in natural rates and policy implications.


Evolving Persistence of Inflation
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
1966
1969
1972
1975
1978
1981
1984
1987
1990
1993
1996
1999
2002
Based on historical data
Based on simulated data absent U* misperceptions

Searching for Robustness

- Estimate a macro model
- Calibrate natural rate (NR) models
- Calibrate a learning model for agents' expectations formation
- Calibrate central bank's model for estimating natural rates
- Analyze monetary policy,
  - with private sector learning, but constant NR
  - with private sector learning and unobserved time-varying NR
- Identify characteristics of policies that are robust to misspecification of expectations and NR

The Model

"Price-setting with indexation"

$$\pi_t = \phi_\pi \pi^e_{t+1} + (1-\phi_\pi)\pi_{t-1} + \alpha_\pi (u_t - u^*_t) + e_{\pi,t}, \quad e_\pi \sim N(0, \sigma^2_{e_\pi}),$$

"IS Curve with habit formation"

$$u_t = \phi_u u^e_{t+1} + (1-\phi_u)u_{t-1} + \alpha_u\,(r^e_t - r^*) + e_{u,t}, \quad e_u \sim N(0, \sigma^2_{e_u}),$$

The Estimated Model

$$\pi_t = 0.5\,\pi^e_{t+1} + 0.5\,\pi_{t-1} \underset{(0.084)}{-0.192} (u_t - u^*_t) + e_{\pi,t}, \quad \hat{\sigma}_{e_\pi} = 1.11$$

$$u_t = 0.5\,u^e_{t+1} + 0.5\,u_{t-1} \underset{(0.017)}{+0.036} (\tilde{r}^e_t - r^*) + e_{u,t}, \quad \hat{\sigma}_{e_u} = 0.29$$

Sample 1981–2003

Method: OLS, using SPF as proxy for expectations

Natural Rates

The natural rate of unemployment (u^*)

$$u_t^* = 0.01\bar{u}^* + 0.99\ u_{t-1}^* + e_{u^*,t},$$

The natural rate of interest (r^*)

$$r_t^* = 0.01\bar{r}^* + 0.99\ r_{t-1}^* + e_{r^*,t},$$

Calibration

$$s = 1: \quad \sigma_{e_{u^*}} = 0.070, \quad \sigma_{e_{r^*}} = 0.085.$$

$$s = 2: \quad \sigma_{e_{u^*}} = 0.140, \quad \sigma_{e_{r^*}} = 0.170.$$

Monetary Policy

Generalized Taylor Rule

$$
\begin{aligned}
i_t \;=\; & \theta_i i_{t-1} + (1-\theta_i)(\hat{r}_t^* + \pi_{t-1}) \\
& + \; \theta_\pi(\pi_{t-1} - \pi^*) + \theta_u(u_{t-1} - \hat{u}_t^*) + \theta_{\Delta u}(u_{t-1} - u_{t-2}),
\end{aligned}
$$

Taylor-style Rule: $i_t = \hat{r}_t^* + \pi_{t-1} + \theta_\pi(\pi_{t-1} - \pi^*) + \theta_u(u_{t-1} - \hat{u}_t^*)$

Difference Rule: $i_t = i_{t-1} + \theta_\pi(\pi_{t-1} - \pi^*) + \theta_{\Delta u}(u_{t-1} - u_{t-2})$

Monetary Policy Objective

Minimize the loss:

$$\mathcal{L} = Var(\pi) + \lambda Var(\tilde{u}) + \nu Var(\Delta(i)),$$

where $VAR(x)$ denotes the unconditional variance of variable x.

Unemployment gap: $\tilde{u}_t = u_t - u_t^*$
Benchmark setting: $\lambda = 4$, $\nu = 0.25$

Private Sector Learning

Agents estimate a restricted VAR that corresponds to the RE solution of the model assuming constant natural rates

$$c_t = c_{t-1} + \kappa R_t^{-1} X_t (Y_t - X_t' c_{t-1}),$$

$$R_t = R_{t-1} + \kappa (X_t X_t' - R_{t-1}),$$

where κ is the constant gain.

$X_t = \{\pi_t, u_t, i_t\}$;

c_t: VAR coefficients

Calibration of κ

Method: Recursively estimate restricted VAR with weighted least squares.

Select κ that yields forecast that best match the SPF forecasts.

Result: $\kappa = 0.02$ yields best fit, but estimated with little precision

Examine range of $\kappa \in \{0.01, 0.02, 0.03\}$ (and perfect knowledge/RE) as alternative models for expectaions formation.

Central Bank Natural Rate Estimates

The natural rate of unemployment ($\hat{u}^*$)

$$\hat{u}_t^* = \hat{u}_{t-1}^* + 0.005(u_{t-1} - \hat{u}_{t-1}^*)$$

The natural rate of interest ($\hat{r}^*$)

$$\hat{r}_t^* = \hat{r}_{t-1}^* + 0.005(r_{t-1} - \hat{r}_{t-1}^*)$$

Simulation Method

Public estimated VAR matrices c and R are initialized to values implied by RE with constant NR

Stochastic simulation of 41,000 periods

First 1000 periods dropped to remove initial conditions, leaving 10,000 years of simulated data

Monetary Policy with Learning and Constant Natural Rates

Natural rates constant at unconditional means.

Policymaker knows unconditional mean of natural rates

Private agents update their forecasting model each period

Performance of the Taylor Rule with Constant Natural Rates

$$i_t = \hat{r}_t^* + \pi_{t-1} + 0.5 \times (\pi_{t-1} - \pi^*) - 1 \times (u_{t-1} - \hat{u}_t^*)$$

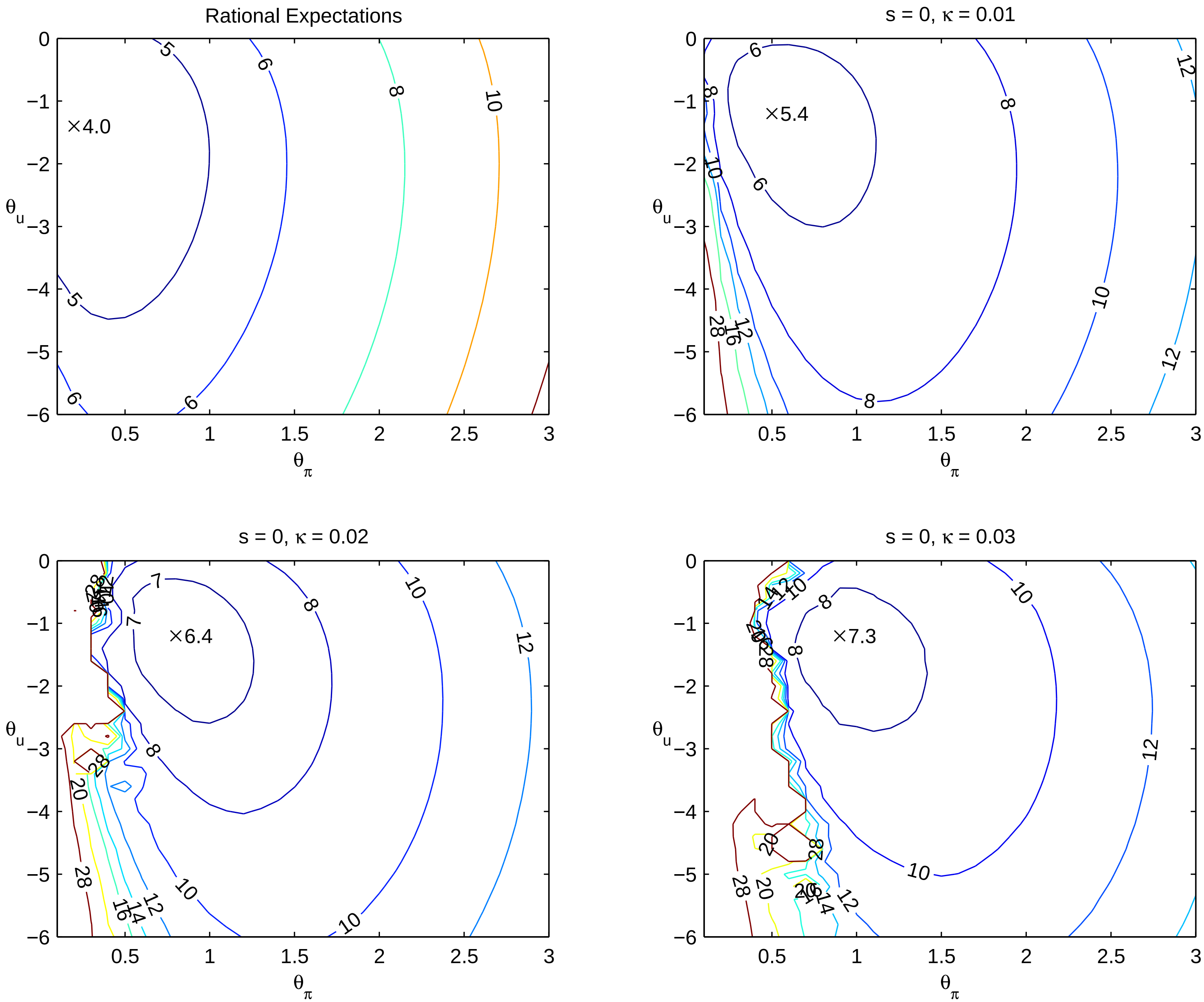
	Standard Deviation			First-order Autocorrelation		
κ	π	$u - u^*$	Δi	π	$u - u^*$	i
RE	1.48	0.54	1.96	0.64	0.77	0.60
0.01	1.68	0.63	1.97	0.72	0.83	0.67
0.02	1.95	0.72	1.99	0.79	0.86	0.75
0.03	2.13	0.79	2.03	0.81	0.88	0.78
0.04	2.30	0.83	2.06	0.84	0.89	0.81

Optimized Taylor Rule with Constant Natural Rates

$$i_t = \hat{r}_t^* + \pi_{t-1} + \theta_\pi(\pi_{t-1} - \pi^*) + \theta_u(u_{t-1} - \hat{u}_t^*)$$

	Policy Rule Coefficient		Standard Deviation			First-order Autocorr.		Loss	
κ	π	$u - u^*$	π	$u - u^*$	Δi	π	$u - u^*$	$\mathcal{L}$	$\mathcal{L}^{RE}$
RE	0.16	-1.37	1.61	0.48	1.52	0.71	0.75	4.0	4.0
0.01	0.53	-1.21	1.69	0.61	2.02	0.73	0.82	5.4	7.0
0.02	0.77	-1.20	1.73	0.72	2.34	0.74	0.85	6.4	10.2
0.03	0.89	-1.38	1.81	0.77	2.52	0.75	0.86	7.3	12.5
0.04	0.99	-1.39	1.90	0.84	2.68	0.77	0.87	8.2	15.2

The Taylor Rule Under Learning



Summary: Monetary Policy with Constant Natural Rates

Optimal policy assuming rational expectations does poor job of stabilizing inflation when private agents are learning.

Optimal policy responds more aggressively to inflation then under RE, while the response to unemployment gap is about the same

The optimal policy restrains the very high degree of serial correlation in inflation that would occur under learning

Monetary Policy with Learning and Time-varying Natural Rates

Natural rates given by AR(1) processes

Two calibrations of innovation variances

Policymaker updates estimates of natural rates using univariate constant gain algorithm each period

Private agents update their forecasting model each period

Performance of the Taylor Rule with Time-varying Natural Rates

$$i_t = \hat{r}_t^* + \pi_{t-1} + 0.5 \times (\pi_{t-1} - \pi^*) - 1 \times (u_{t-1} - \hat{u}_t^*)$$

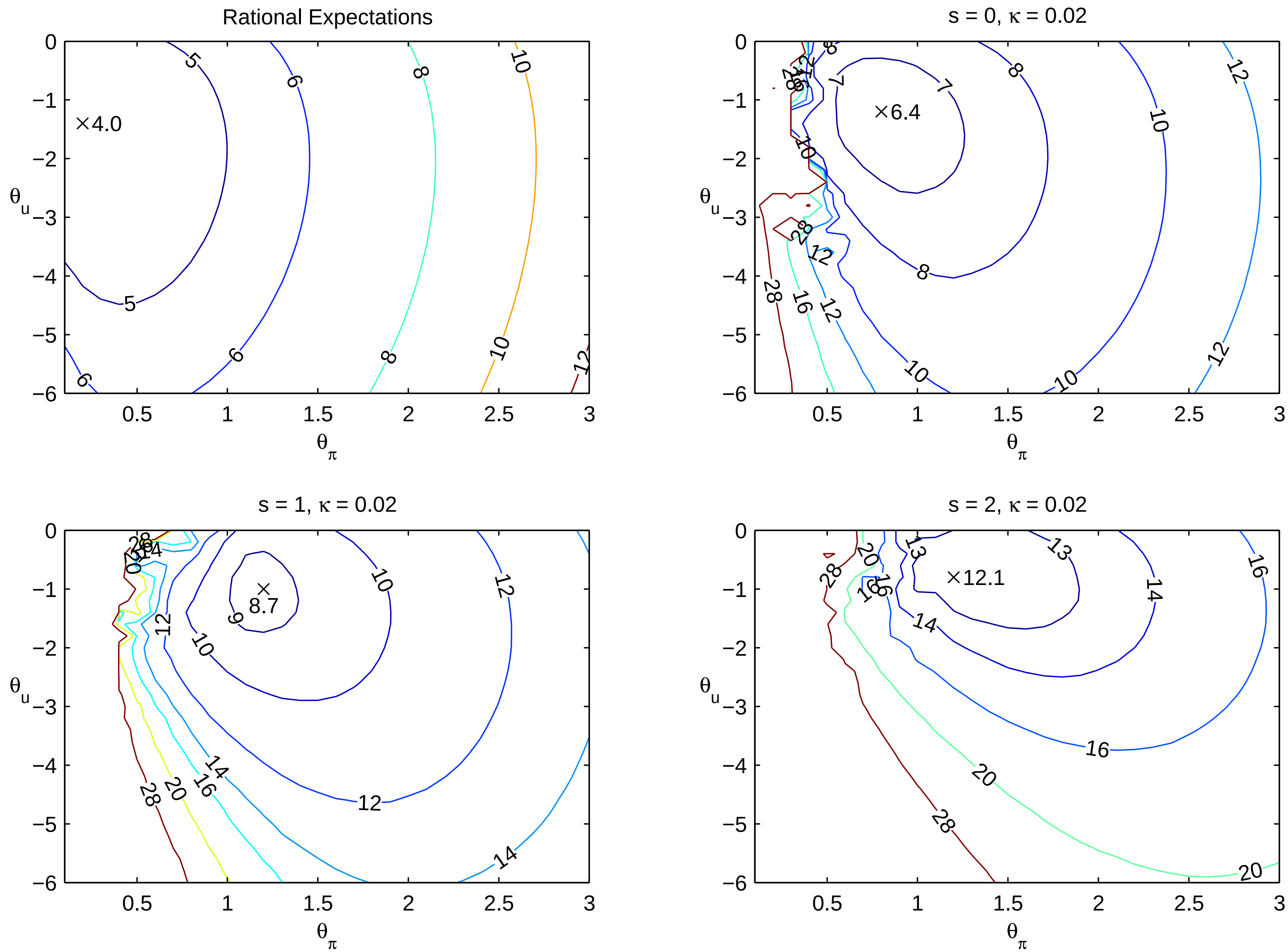
	Standard Deviation			First-order Autocorr.		
κ	π	$u - u^*$	Δi	π	$u - u^*$	i
s = 0						
RE	1.48	0.54	1.96	0.64	0.77	0.60
0.01	1.68	0.63	1.97	0.72	0.83	0.67
0.02	1.95	0.72	1.99	0.79	0.86	0.75
0.03	2.13	0.79	2.03	0.81	0.88	0.78
0.04	2.30	0.83	2.06	0.84	0.89	0.81
s = 1						
0.01	2.12	0.84	1.99	0.82	0.90	0.82
0.02	2.46	0.90	2.01	0.86	0.91	0.85
0.03	2.61	0.94	2.03	0.88	0.91	0.86
0.04	2.76	0.98	2.06	0.89	0.92	0.87
s = 2						
0.01	2.72	1.14	1.99	0.89	0.94	0.89
0.02	3.14	1.18	2.01	0.92	0.94	0.91
0.03	3.34	1.20	2.03	0.92	0.94	0.91
0.04	3.48	1.22	2.09	0.93	0.94	0.92

Optimized Taylor Rule with Time-varying Natural Rates

$$i_t = \hat{r}_t^* + \pi_{t-1} + \theta_\pi(\pi_{t-1} - \pi^*) + \theta_u(u_{t-1} - \hat{u}_t^*)$$

	Policy Rule Coefficient		Standard Deviation			Loss	
κ	π	$u - u^*$	π	$u - u^*$	Δi	$\mathcal{L}_2$	$\mathcal{L}_2^{RE}$
s = 0							
RE	0.16	-1.37	1.60	0.47	1.52	4.0	4.0
0.01	0.53	-1.21	1.69	0.61	2.02	5.4	7.0
0.02	0.77	-1.20	1.73	0.72	2.34	6.4	10.2
0.03	0.88	-1.38	1.81	0.77	2.52	7.3	12.5
0.04	0.98	-1.39	1.90	0.84	2.68	8.2	15.2
s = 1							
0.01	0.72	-0.57	1.81	0.89	2.27	7.8	14.0
0.02	1.07	-0.99	1.82	0.93	2.75	8.7	21.3
0.03	1.14	-0.97	1.89	0.98	2.87	9.5	24.7
0.04	1.21	-1.18	1.97	1.00	2.99	10.1	28.0
s = 2							
0.01	1.04	-0.32	1.88	1.23	2.71	11.4	25.9
0.02	1.21	-0.67	1.98	1.23	2.94	12.1	34.2
0.03	1.42	-0.89	2.02	1.23	3.26	12.8	39.1
0.04	1.39	-1.02	2.15	1.23	3.24	13.3	40.6

The Taylor Rule Under Learning with Time-Varying Natural Rates



Optimized Difference Rules with Time-varying Natural Rates

$$i_t = i_{t-1} + \theta_\pi(\pi_{t-1} - \pi^*) + \theta_{\Delta u}(u_{t-1} - u_{t-2})$$

	Policy Coefficient		Standard Deviation			Loss	
κ	π	Δu	π	$u - u^*$	Δi	$\mathcal{L}_D^*$	$\mathcal{L}_2^*$
s = 0							
RE	0.31	-3.76	1.75	0.58	1.34	4.9	4.0
0.01	0.49	-4.04	1.79	0.72	1.54	5.9	5.4
0.02	0.55	-4.07	1.86	0.81	1.59	6.7	6.4
0.03	0.76	-4.51	1.85	0.92	1.88	7.7	7.3
0.04	0.69	-4.79	2.01	0.96	1.92	8.6	8.2
s = 1							
0.01	0.54	-3.94	1.78	0.87	1.54	6.8	7.8
0.02	0.66	-4.20	1.83	0.94	1.70	7.6	8.7
0.03	0.85	-4.54	1.83	1.02	1.95	8.5	9.5
0.04	0.95	-4.92	1.88	1.08	2.14	9.4	10.1
s = 2							
0.01	0.68	-3.82	1.78	1.15	1.59	9.1	11.4
0.02	0.84	-4.15	1.81	1.19	1.82	9.8	12.1
0.03	0.93	-4.18	1.85	1.24	1.91	10.5	12.8
0.04	1.02	-4.38	1.87	1.28	2.05	11.2	13.3

Summary: Monetary Policy with Time-Varying NRs

The optimal policy under RE performs very poorly under imperfect knowledge.

More generally, policy rules that respond only modestly to inflation fail to anchor adequately inflation expectations and stabilize inflation, even if they satisfy the Taylor Principle.

For a given κ, time-variation in natural rates raises the optimal policy response to inflation and, in the case of a Taylor-style rule, lowers the response to the perceived unemployment gap.

Robust Policy

Can one rule perform well across the range of plausible models of expectations formation and natural rates?

The optimal policy under RE fails this test.

But, if the models are reasonably "fault tolerant" within a range of values of policy coefficients, a robust policy exists.

Robustness of Optimal Bayesian Rules

Optimize rule with flat prior over a reasonable range of models for expectations formation and natural rates?

—Generalized Rule (4 parameters)
—Level Rule (2 parameters)
—Difference Rule (2 parameters)

$$\mathcal{L}_4^B\text{: } i_t = 0.96 i_{t-1} + (1-0.96)(\hat{r}_t^* + \pi_{t-1}) + 0.69(\pi_{t-1} - \pi^*)$$
$$-0.75(u_{t-1} - \hat{u}_t^*) - 2.58(u_{t-1} - u_{t-2})$$

$$\mathcal{L}_T^B\text{: } i_t = \hat{r}_t^* + \pi_{t-1} + 1.05(\pi_{t-1} - \pi^*) - 0.75(u_{t-1} - \hat{u}_t^*)$$

$$\mathcal{L}_D^B\text{: } i_t = i_{t-1} + 0.65(\pi_{t-1} - \pi^*) - 4.12(u_{t-1} - u_{t-2})$$

Robustness of Optimal Bayesian Generalized Rule

	Standard Deviation			Loss	
κ	π	$u-u^*$	Δi	$\mathcal{L}_4^B$	$\mathcal{L}_4^*$
s = 0					
RE	1.43	0.62	1.48	4.1	3.3
0.01	1.51	0.71	1.50	4.8	4.4
0.02	1.59	0.78	1.52	5.5	5.2
0.03	1.70	0.85	1.54	6.4	6.0
s = 1					
RE	1.51	0.64	1.48	4.4	4.2
0.01	1.55	0.85	1.50	5.8	5.7
0.02	1.64	0.90	1.52	6.4	6.5
0.03	1.77	0.96	1.55	7.3	7.3
s = 2					
RE	1.71	0.67	1.49	5.2	5.0
0.01	1.70	1.11	1.51	8.4	8.2
0.02	1.84	1.14	1.53	9.1	9.0
0.03	1.99	1.18	1.57	10.0	9.8

Robustness of Alternative Bayesian Rules

Loss	$\mathcal{L}_T^B$	$\mathcal{L}_D^B$	$\mathcal{L}_4^B$	$\mathcal{L}_4^*$
s = 0				
RE	5.3	5.2	4.1	3.3
0.01	6.6	6.0	4.8	4.4
0.02	7.2	6.8	5.5	5.2
0.03	8.1	7.7	6.4	6.0
s = 1				
RE	5.6	5.3	4.4	4.2
0.01	8.1	6.8	5.8	5.7
0.02	8.8	7.6	6.4	6.5
0.03	10.1	8.5	7.3	7.3
s = 2				
RE	6.6	5.7	5.2	5.0
0.01	12.2	9.1	8.4	8.2
0.02	12.9	9.9	9.1	9.0
0.03	13.3	10.7	10.0	9.8

Conclusion

- Imperfect knowledge is a first order problem for policy design.
- With imperfect knowledge, policies that would be optimal under the assumption of rational expectations can do a horrible job stabilizing inflation and unemployment.
- The design of robust rules appears possible.
  - Level-style Taylor rules are not robust.
  - Difference rules are more robust.
  - Generalized rules that combine elements of Taylor-style rules and difference rules perform even better.
- Robust policies exhibit inertial behavior and place much greater emphasis on price stability than is often suggested as being optimal in more simplistic optimal policy experiments.

Optimized Generalized Rule with Time-varying NRs

	Policy Rule Coefficient				Standard Deviation			Loss		
κ	i	π	$u-u^*$	Δu	π	$u-u^*$	Δi	$\mathcal{L}_4^*$	$\mathcal{L}_D^*$	$\mathcal{L}_2^*$
s = 0										
RE	0.76	0.18	-0.97	-0.43	1.51	0.47	0.77	3.3	4.9	4.0
0.01	0.85	0.42	-1.01	-1.23	1.60	0.61	1.12	4.4	5.9	5.4
0.02	0.87	0.61	-1.09	-1.78	1.64	0.71	1.43	5.2	6.7	6.4
0.03	0.86	0.64	-1.15	-2.19	1.77	0.75	1.59	6.0	7.7	7.3
0.04	0.92	0.76	-1.36	-2.23	1.83	0.82	1.74	6.8	8.6	8.2
s = 1										
0.01	0.95	0.52	-0.74	-1.89	1.60	0.83	1.21	5.7	6.8	7.8
0.02	0.95	0.66	-0.78	-2.43	1.66	0.89	1.47	6.5	7.6	8.7
0.03	0.99	0.84	-0.95	-3.01	1.71	0.95	1.80	7.3	8.5	9.5
0.04	0.97	1.00	-1.25	-2.92	1.77	0.99	2.04	8.1	9.4	10.1
s = 2										
0.01	1.00	0.68	-0.62	-2.50	1.64	1.12	1.41	8.2	9.1	11.4
0.02	1.00	0.85	-0.60	-3.04	1.67	1.17	1.70	9.0	9.8	12.1
0.03	1.00	0.97	-0.71	-3.27	1.74	1.21	1.91	9.8	10.5	12.8
0.04	1.00	0.96	-0.76	-3.35	1.82	1.23	1.94	10.3	11.2	13.3

Robust Policy

Can one rule perform well across the range of plausible models of expectations formation and natural rates?

The optimal policy under RE fails this test.

But, if the models are reasonably "fault tolerant" within a range of values of policy coefficients, a robust policy exists.

Robustness of Benchmark ($\kappa = 0.02, s = 1$) Generalized Policy Rule

	Standard Deviation			Loss	
κ	π	$u - u^*$	Δi	$\mathcal{L}$	$\mathcal{L}_4^*$
$s = 0$					
RE	1.43	0.61	1.43	4.0	3.3
0.01	1.51	0.69	1.46	4.7	4.4
0.02	1.60	0.76	1.47	5.4	5.2
0.03	1.72	0.83	1.50	6.3	6.0
0.04	1.84	0.89	1.54	7.2	6.8
$s = 1$					
0.01	1.55	0.84	1.45	5.8	5.7
0.02	1.66	0.89	1.47	6.5	6.5
0.03	1.79	0.95	1.51	7.4	7.3
0.04	1.92	1.02	1.54	8.4	8.1
$s = 2$					
0.01	1.73	1.11	1.46	8.5	8.2
0.02	1.89	1.14	1.48	9.3	9.0
0.03	2.02	1.17	1.53	10.2	9.8
0.04	2.12	1.20	1.56	10.9	10.3

Conclusion

With imperfect knowledge, policies that would be optimal under the assumption of rational expectations can do a horrible job stabilizing inflation and unemployment.

Generalized rules that combine elements of Taylor-style rules and difference rules appear more robust in the presence of unknown natural rates and imperfections in expectations formation.

Our benchmark generalized policy rule is nearly completely inertial and performs very well across a wide range of specifications of private-sector learning and natural rate variation.

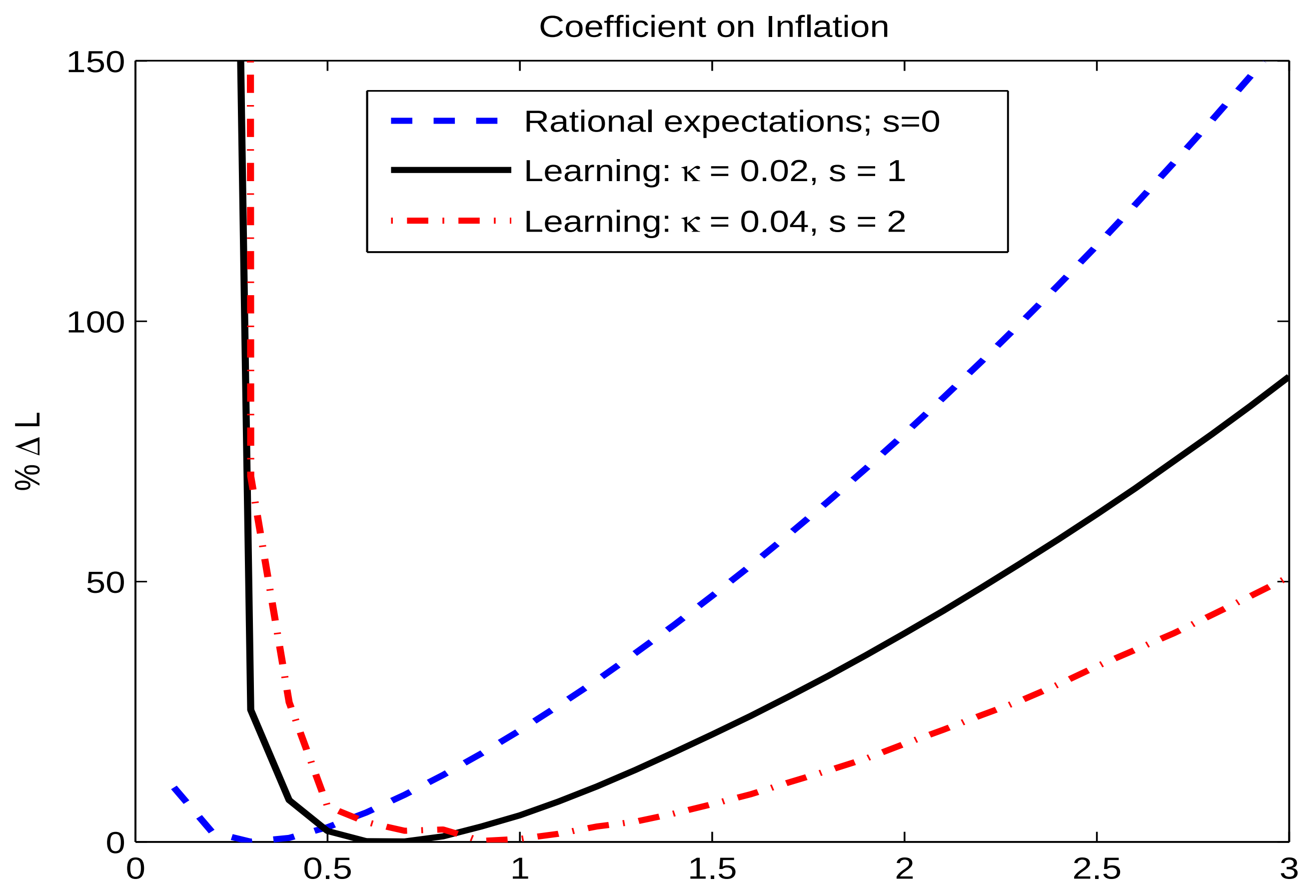
Fault Tolerance

To illuminate the reason why the optimal policy under rational expectations performs poorly, we examine the "fault tolerance" of the model for difference rules and Taylor-style rules (Levin and Williams 2003).

We trace out the loss for different values of θ_π, holding other coefficients in the policy rule fixed at their respective optimal values.

We repeat this exercise for three calibrations of the learning model and natural rate processes: $(RE, s = 0), (\kappa = 0.02, s = 1), (\kappa = 0.04, s = 2)$.

Fault Tolerance: Difference Rule



Fault Tolerance: Taylor-style Rule