Analysing Productivity Cycles in the Euro area, US and UK Using Wavelet Analysis

Patrick Crowley, Douglas Maraun and David Mayes

November 2005

1 Introduction

Productivity usually thought of in terms of shocks - this comes from real business cycle theory approach;

But actually little empirical analysis on productivity as a variable;

Perhaps reason is that productivity is a derived variable - either as simple calculation or Solow residual;

Leads to several questions:

- Can we identify cycles in productivity?

- If so, at what frequencies are these productivity cycles?

- Are these growth frequencies similar between the Euro area, US and UK?

In this paper we use 3 variations of wavelet analysis to analyse these questions

Why wavelets?

i) wavelets are both time and frequency resolved (time-frequency analysis);

ii) wavelets are a fairly common way to analyse time series in other disciplines (signal processing, meteorology, astronomy, medical science, engineering) - so lots of different techniques available. Here we use a) MODWT, b) CWT and c) MODWT with HWP;

iii) recent developments allow extraction of cycles at various frequencies so that further analysis can be conducted - so in a way this is just a first step

Useful references for wavelet analysis with applications in economics are Ramsay (1998), Percival and Waldon (2000), Gencay, Selcuk and Whitcher (2001) and Crowley (2005). This work can be seen as complementary to another paper by Crowley, Maraun and Mayes (2005) which looks at growth cycles in Germany, France and Italy.

Data

- Uses quarterly real seasonally adjusted GDP data from early 1970s for US, Euro area, and UK;

- Percentage annual change

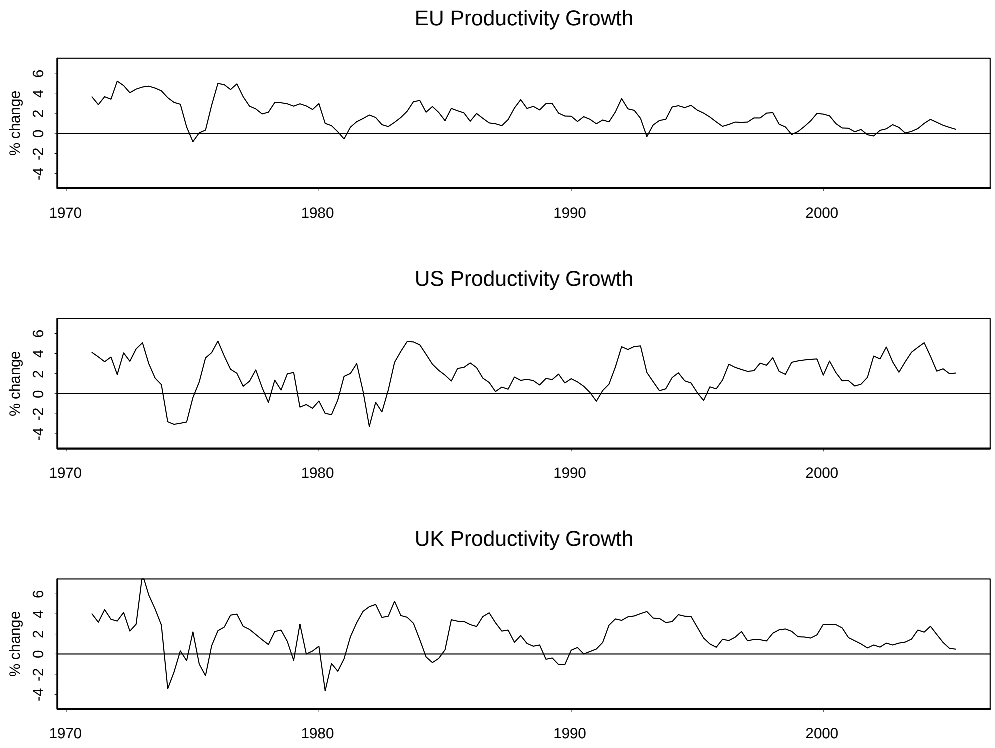


Figure 1: Percentage change in year-over-year quarterly productivity data

2 Scale decomposition by MODWT

2.1 Methodology

Assume we are dealing with symmlets:

$$\phi_{j,k} = 2^{-\frac{j}{2}}\phi(\frac{t-2^{j}k}{2^{j}}) \tag{1}$$

$$\psi_{j,k} = 2^{-\frac{j}{2}}\psi(\frac{t-2^{j}k}{2^{j}}) \tag{2}$$

where j indexes the scale, and k indexes the translation. If the wavelet coefficients are approximately given by the integrals:

$$s_{J,k} \approx \int x(t)\phi_{J,k}(t)dt \tag{3}$$

$$d_{j,k} \approx \int x(t)\psi_{j,k}(t)dt \tag{4}$$

$j = 1, 2, ...J$ such that J is the maximum scale sustainable with the data to hand then a multiresolution decomposition (MRD) of $x(t)$ is given by:

$$x(t) = \sum_k s_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \sum_k d_{J-1,k}\psi_{J-1,k}(t) + ... + \sum_k d_{1,k}\psi_{1,k}(t) \quad (5)$$

Interpretation of crystals is important here:

Scale crystals	Quarterly frequency resolution
d1	1-2
d2	2-4
d3	4-8=1-2yrs
d4	8-16=2-4yrs
d5	16-32=4-8ys
d6	64-128=8-16yrs
d7	128-256=16-32yrs
d8	etc

Table 1: Frequency interpretation of MRD scale levels

MODWT gives up the orthogonality property of the DWT to gain other features, such as given in as:

- the ability to handle any sample size regardless of whether dyadic or not;
- increased resolution at coarser scales as the MODWT oversamples the data;
- translation-invariance - in other words the MODWT crystal coefficients do not change if the time series is shifted in a "circular" fashion; and
- the MODWT produces a more asymptotically efficient wavelet variance estimator than the DWT.

2.2 Empirical results

2.2.1 MODWT results

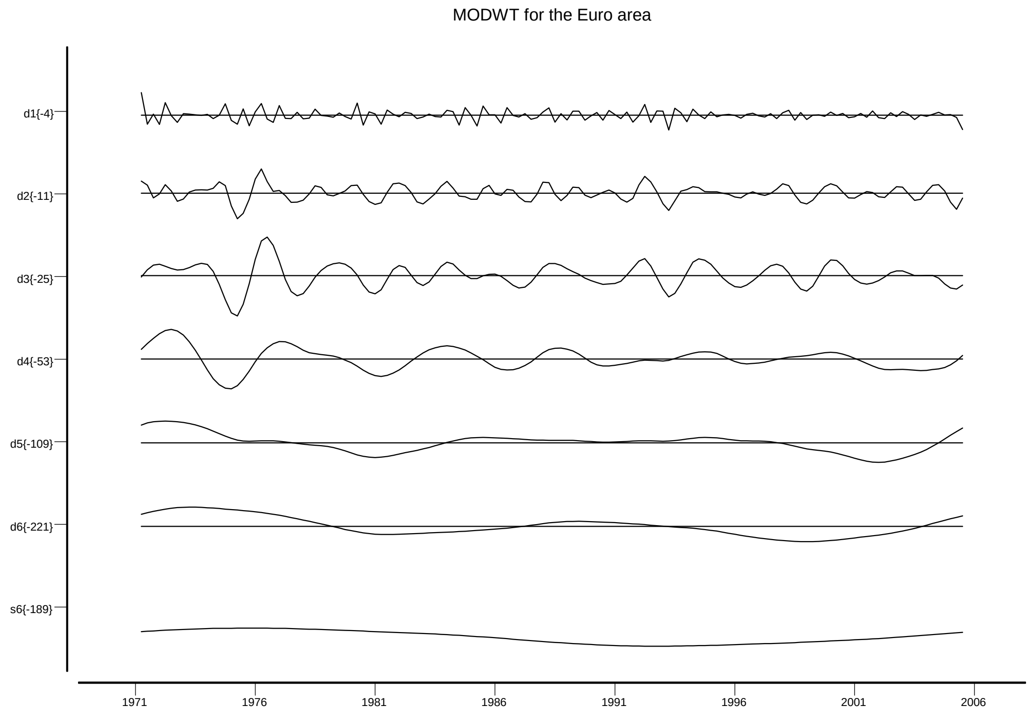


Figure 2: MODWT for Euro area productivity

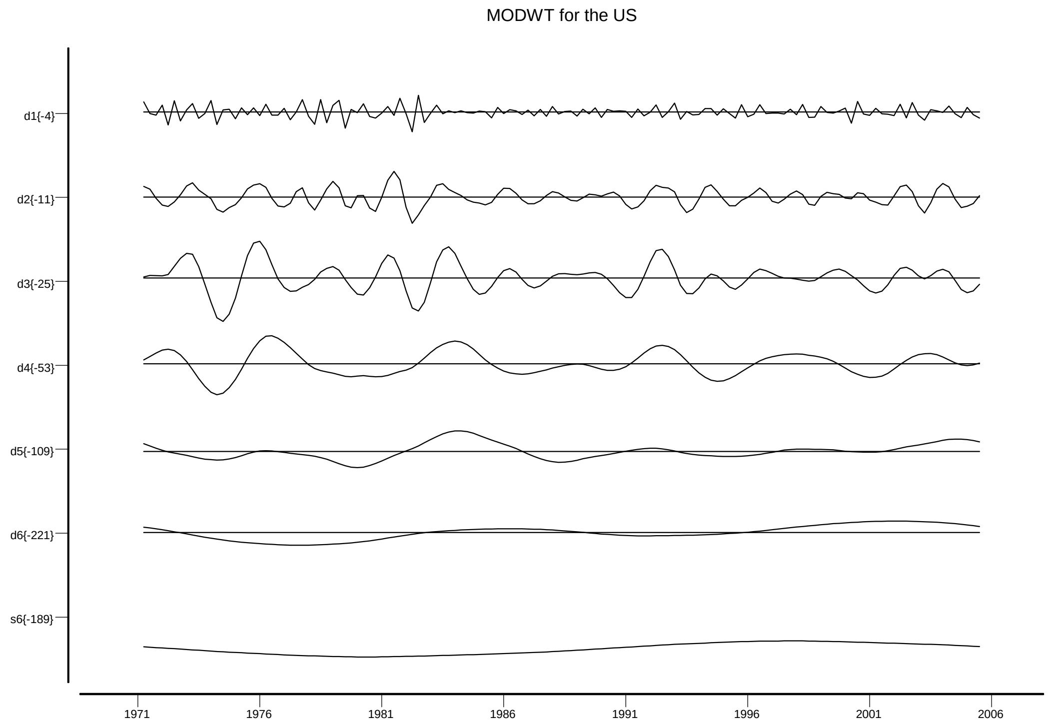


Figure 3: MODWT for US productivity

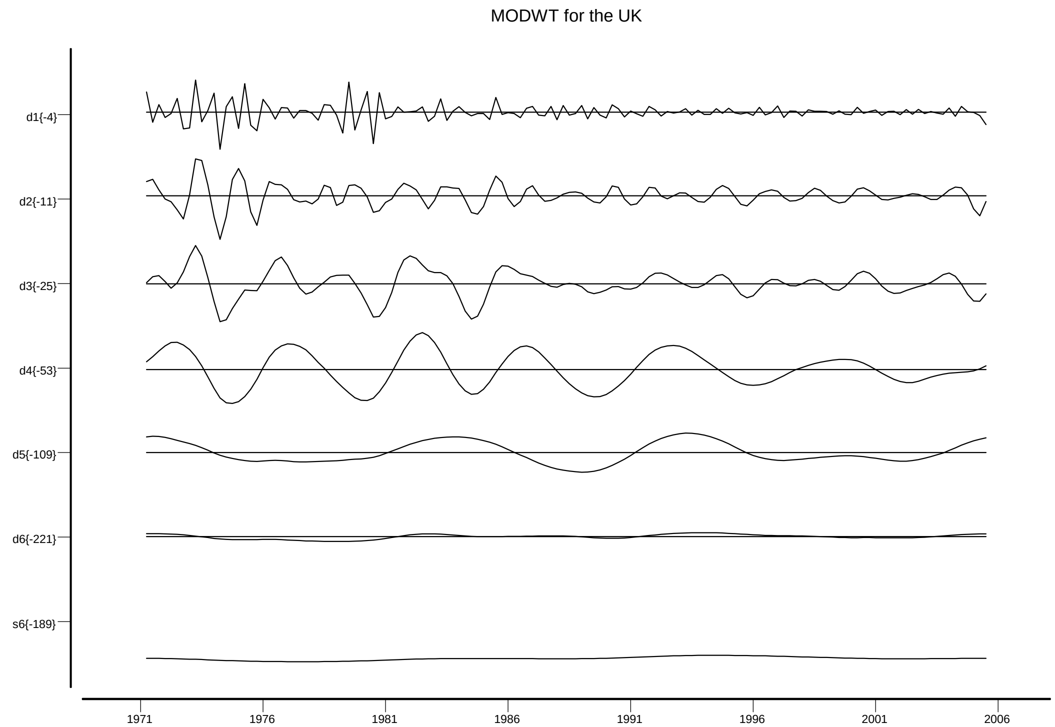


Figure 4: MODWT for UK productivity

Observations:

i) Crystal d1 appears in all cases to exhibit some volatility, but since the 1970s, the volatility at this frequency has tended to be low;

ii) In all 3 cases, crystals d2 and d3 appear to have roughly the same pattern, suggesting that just one cycle is at work at these frequencies (of 2-8 quarters);

iii) In all 3 cases crystal d4 (2-4yrs) appears to have a fairly regular cycle, although this crystal appears to be stronger for the US and UK than it does for the Euro area. Further d5 appears to follow a similar pattern here for the Euro area and the US, suggesting a 2-8yrs cycle, which accords with business cycle frequencies;

iv) In all cases their appears to be little activity in the d6 crystal (8-16yrs) although in all cases there appears to be a residual cycle in the s6 crystal (at a frequency of approximately 20yrs, given the half cycles that we observe in the wavelet smooth). The wavelet smooth does appear to match the general perceptions about productivity trends in each of the three cases considered here.

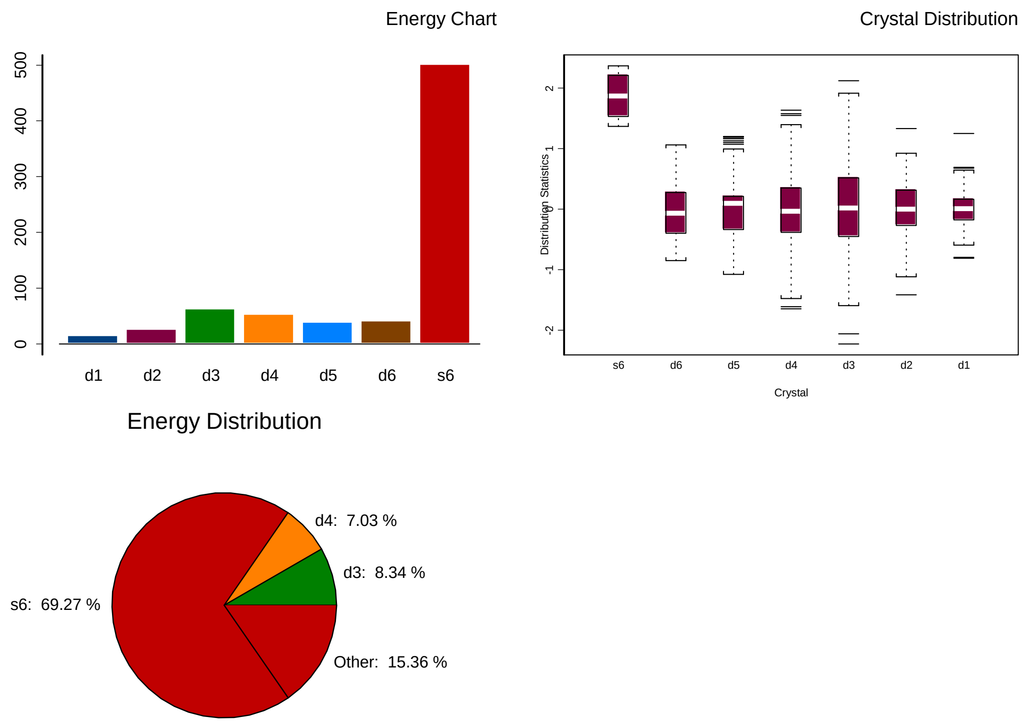


Figure 5: Euro area crystal energy distribution

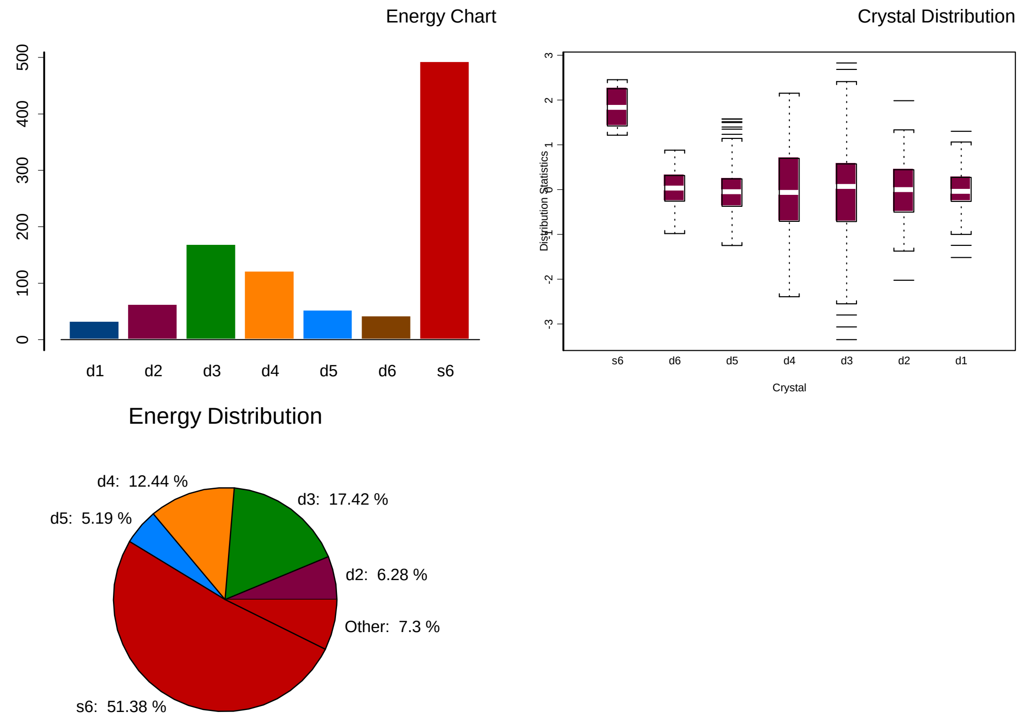


Figure 6: US crystal energy distribution

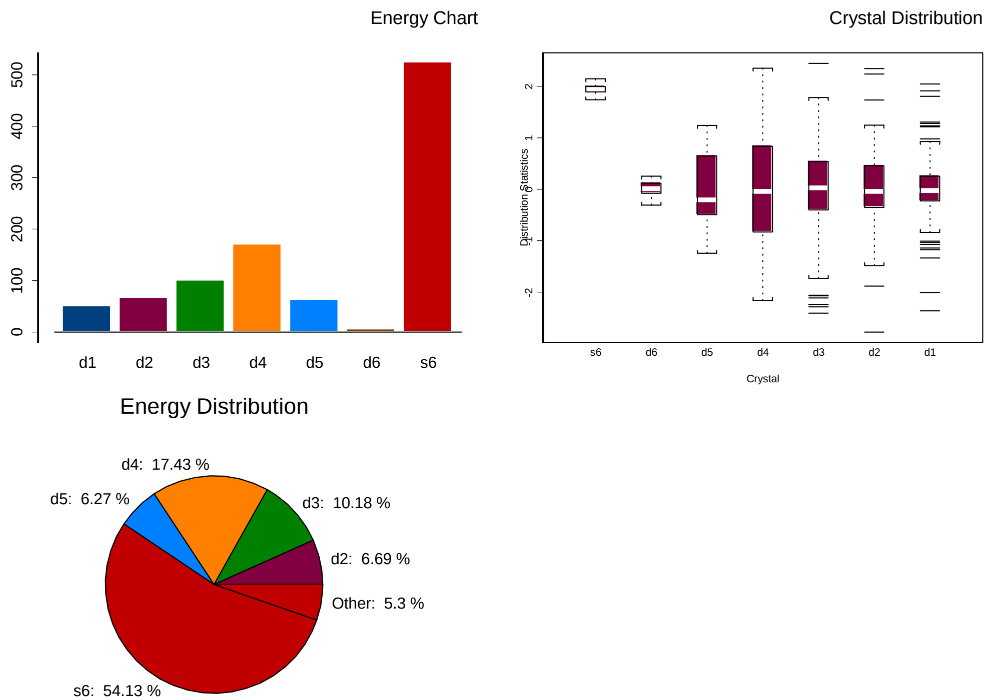


Figure 7: UK crystal energy distribution

2.2.2 Correlations

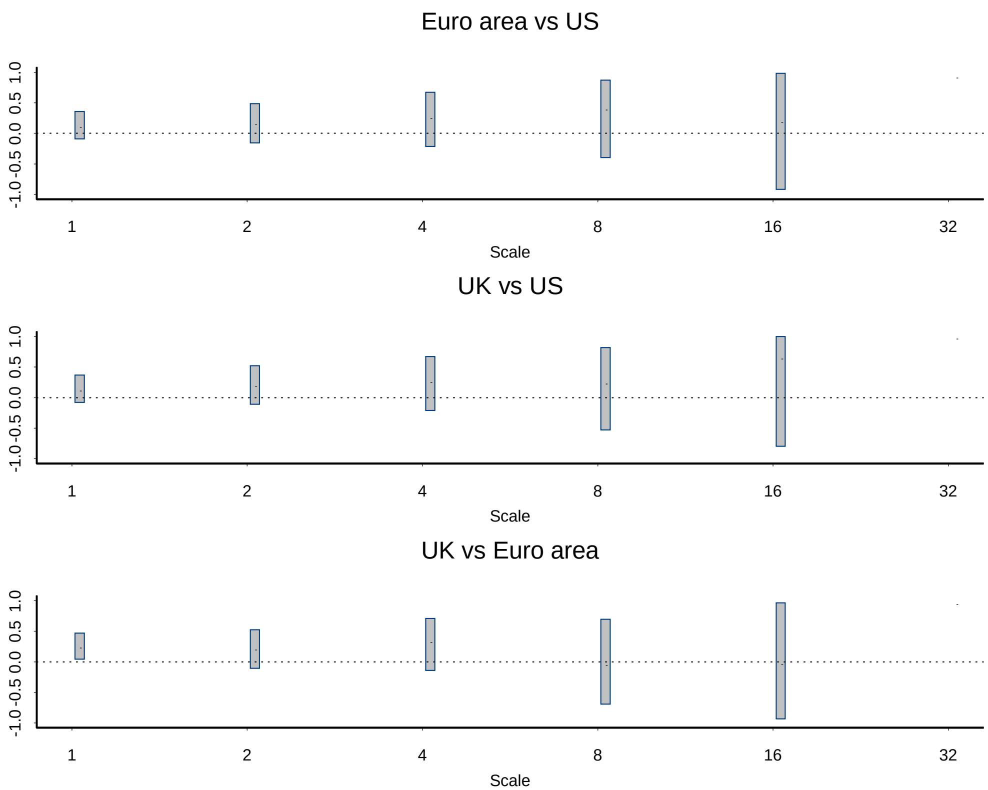


Figure 8: Wavelet correlations with 95% confidence intervals

Cross-correlations

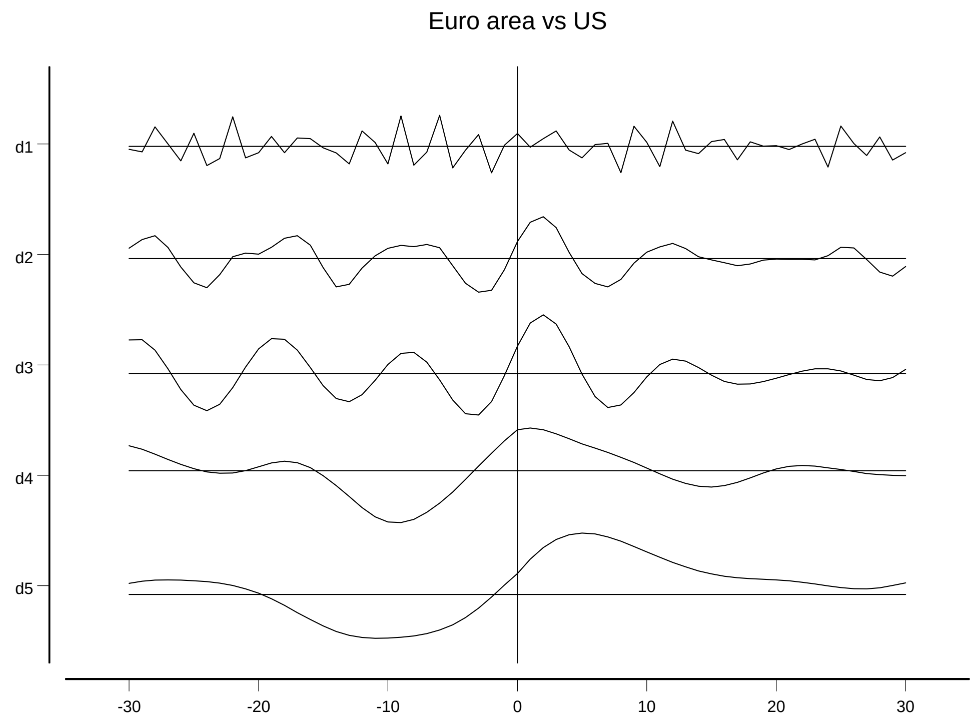


Figure 9: Cross-correlation plot for Euro area vs US productivity

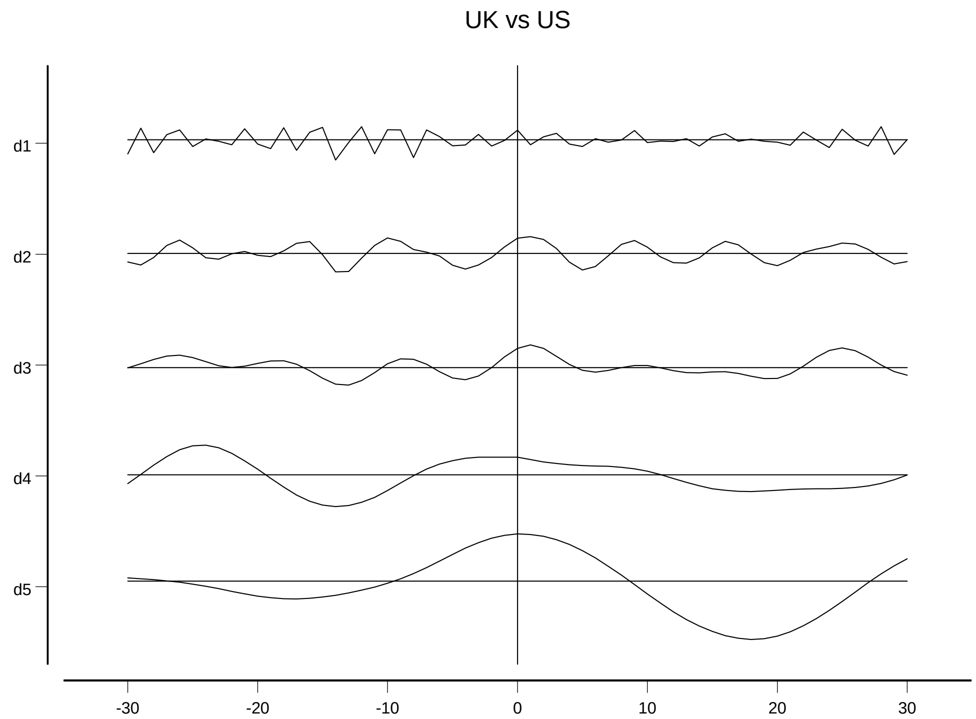


Figure 10: Cross-correlation plot for UK vs US productivity

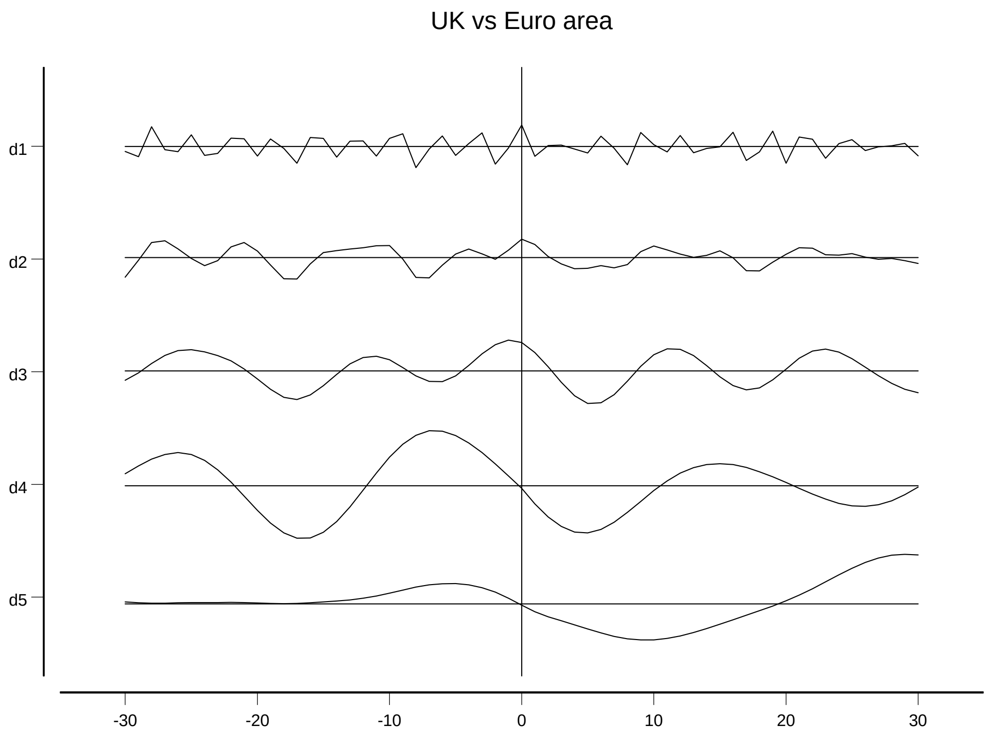


Figure 11: Cross correlation plot for UK vs Euro area productivity

3 Continuous Wavelet Transforms (CWTs)

3.1 Methodology

- Uses Morlet wavelet, which is defined as:

$$\psi(\theta) = \pi^{-0.25} e^{i\omega\pi} e^{-\frac{\pi^2}{2}}$$

which is a symmetric wavelet

- given a time series $x(t)$ and an analysing wavelet function $\psi(\theta)$, then the continuous wavelet transformation (CWT) is given by:

$$W(t, s) = \int_{-\infty}^{\infty} \frac{d\tau}{s^{\frac{1}{2}}} \psi^* \left(\frac{\tau - t}{s} \right) x(\tau) \tag{6}$$

- For a discrete numerical evaluation using frequency domain notation, we get:

$$W_k(s) = \sum_{k=0}^{N} s^{\frac{1}{2}} \widehat{x}_t \widehat{\psi}^*(s\omega_k) e^{i\omega_k t \partial t} \tag{7}$$

where $\widehat{x}_k$ is the discrete Fourier transform of x_t

- Power spectrum for given scale s at time t

$$WPS(t,s) = E\{W(t,s)W(t,s)^*\}$$

- Cross wavelet power spectrum given two variables, x and y:

$$WCS^{xy}(t,s) = E\{W^x(t,s)W^y(t,s)^*\}$$

- Coherence

$$WCO^{xy}(t,s) = \frac{|WCS^{xy}(t,s)|}{E\{[W^x(t,s)W^y(t,s)^*]^{\frac{1}{2}}\}}$$

- Phase function $\Phi(s)$ given from

$$WCS^{xy}(t,s) = |WCS^{xy}(t,s)|\, e^{i\Phi(s)}$$

3.2 Results

3.2.1 Power spectra

Here use red noise (AR1) spectra for background and Monte-Carlo generated 90 & 95% significance levels:

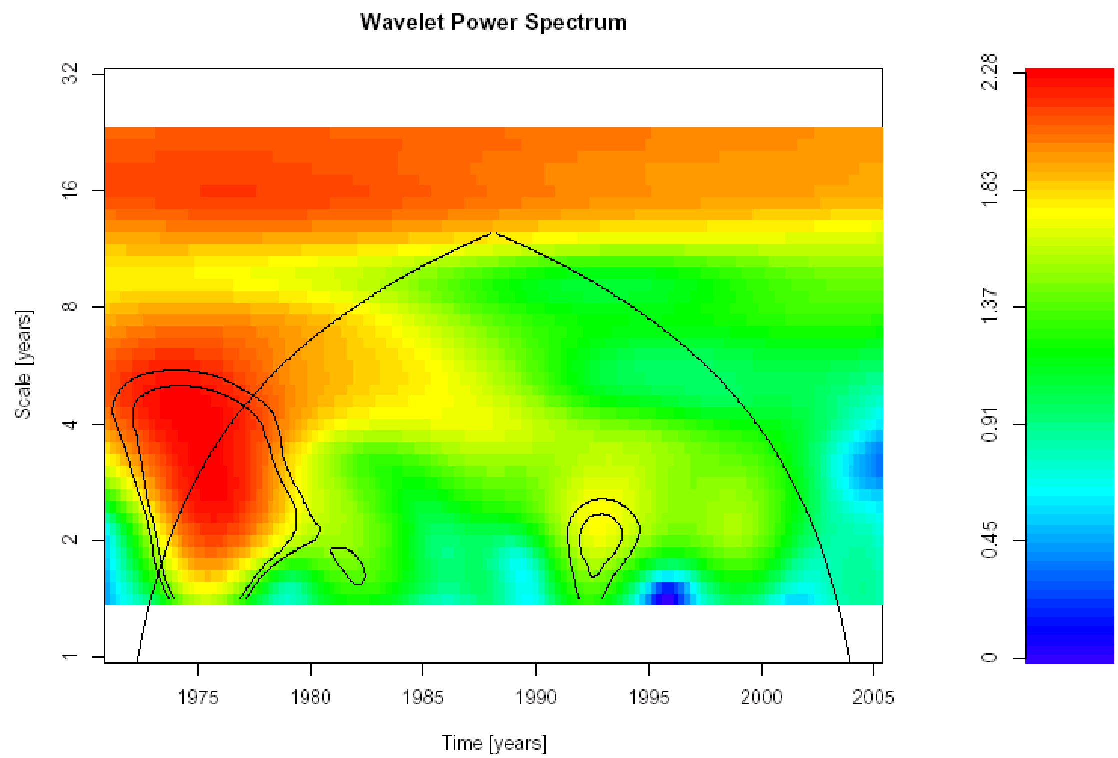


Figure 12: Power spectrum for Euro area productivity

Figure 13: Power spectrum for US productivity

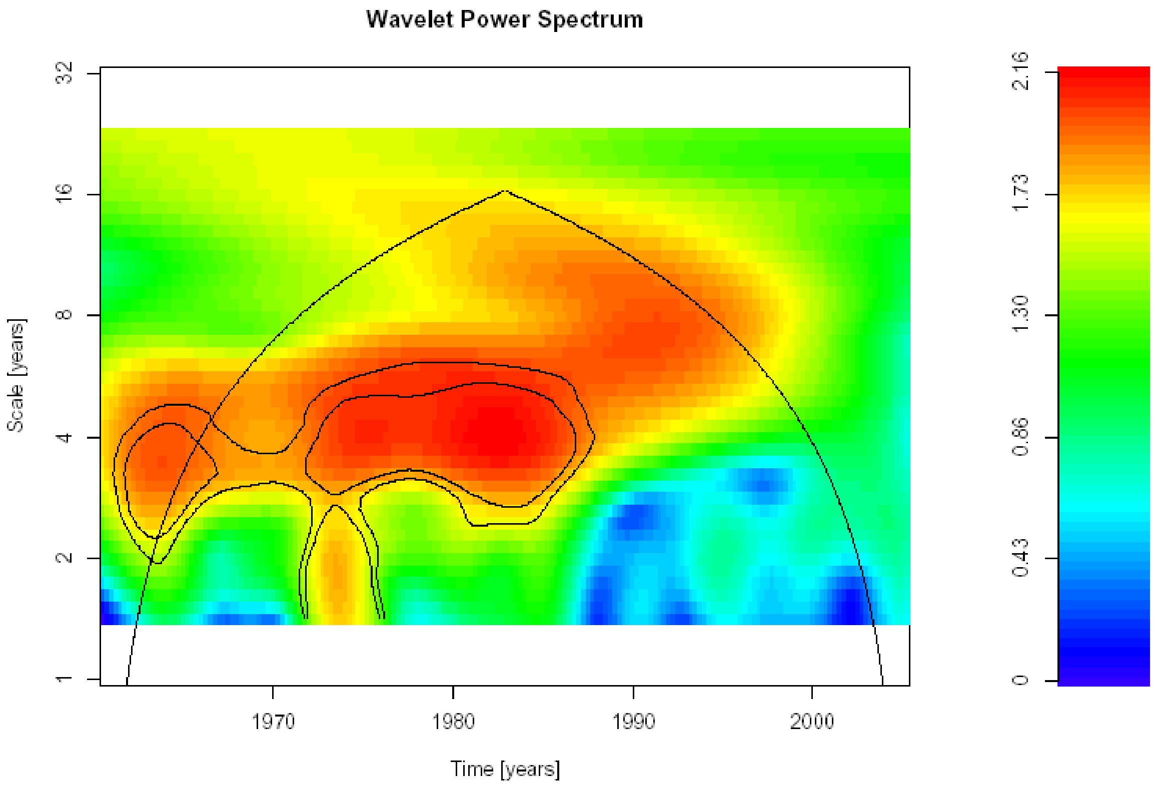


Figure 14: Power spectrum for UK productivity

3.2.2 Coherence and phase

No specific background spectra necessary or assumed here - test is for zero coherence

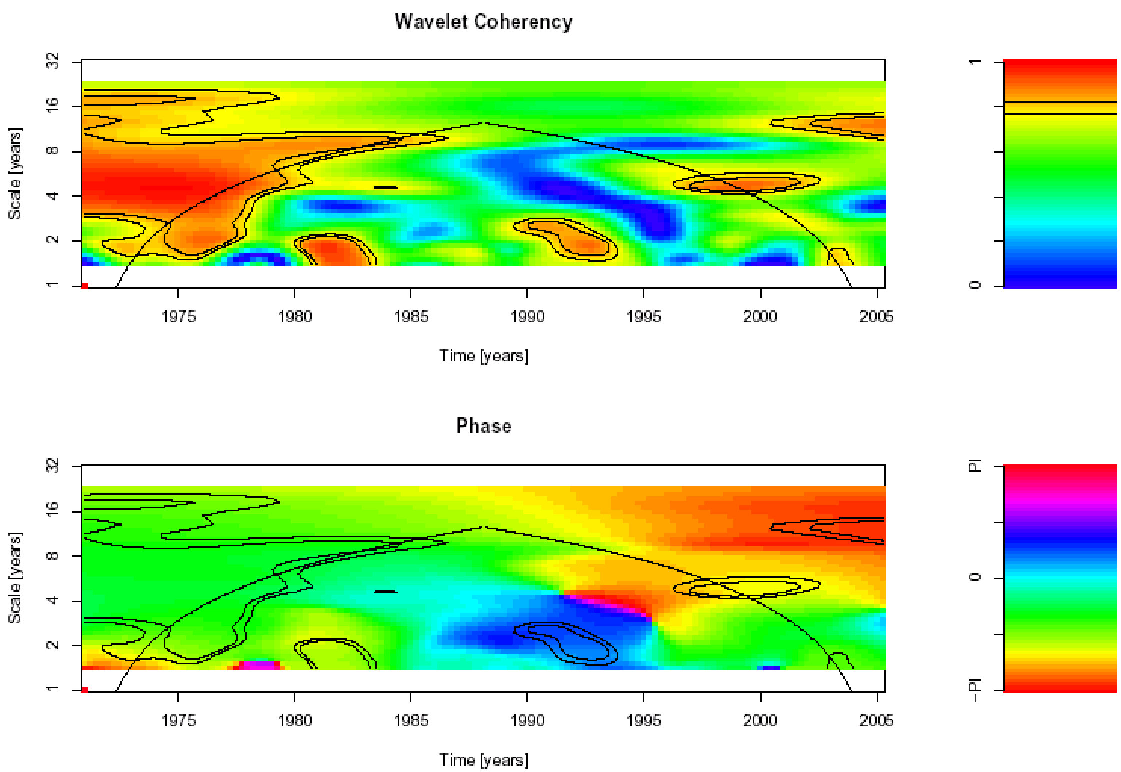


Figure 15: CWT for Euro area vs US productvity

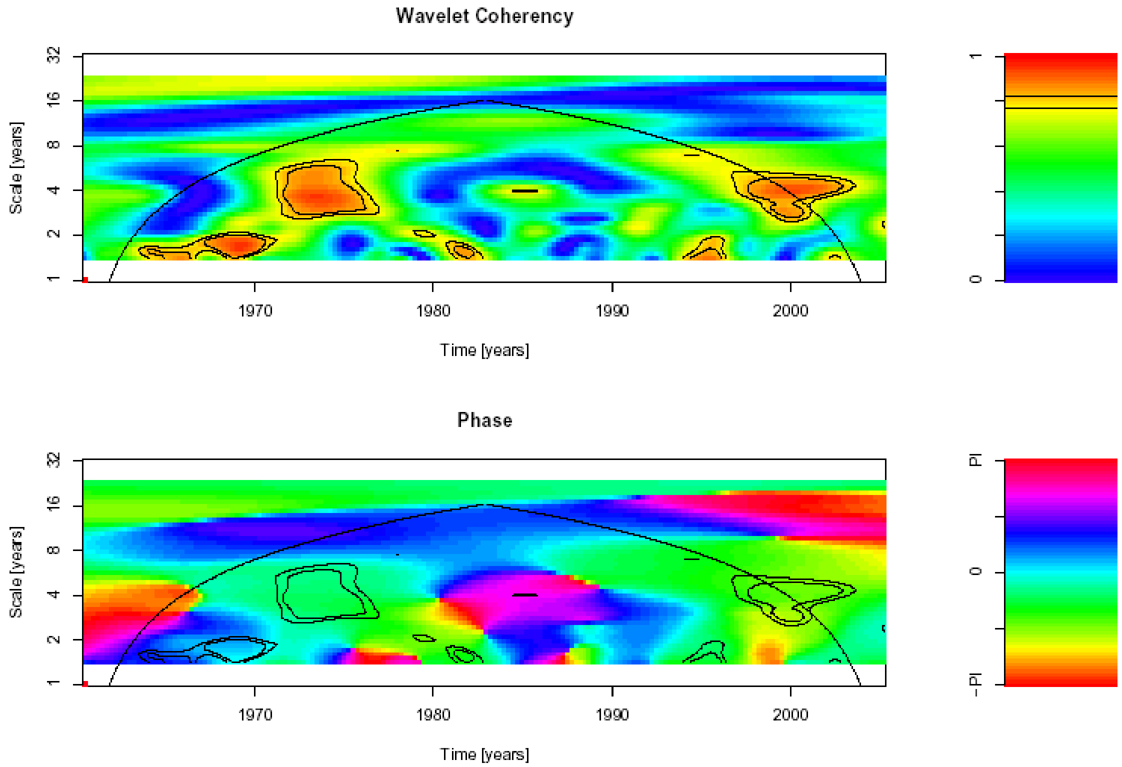


Figure 16: CWT for UK vs US productivity

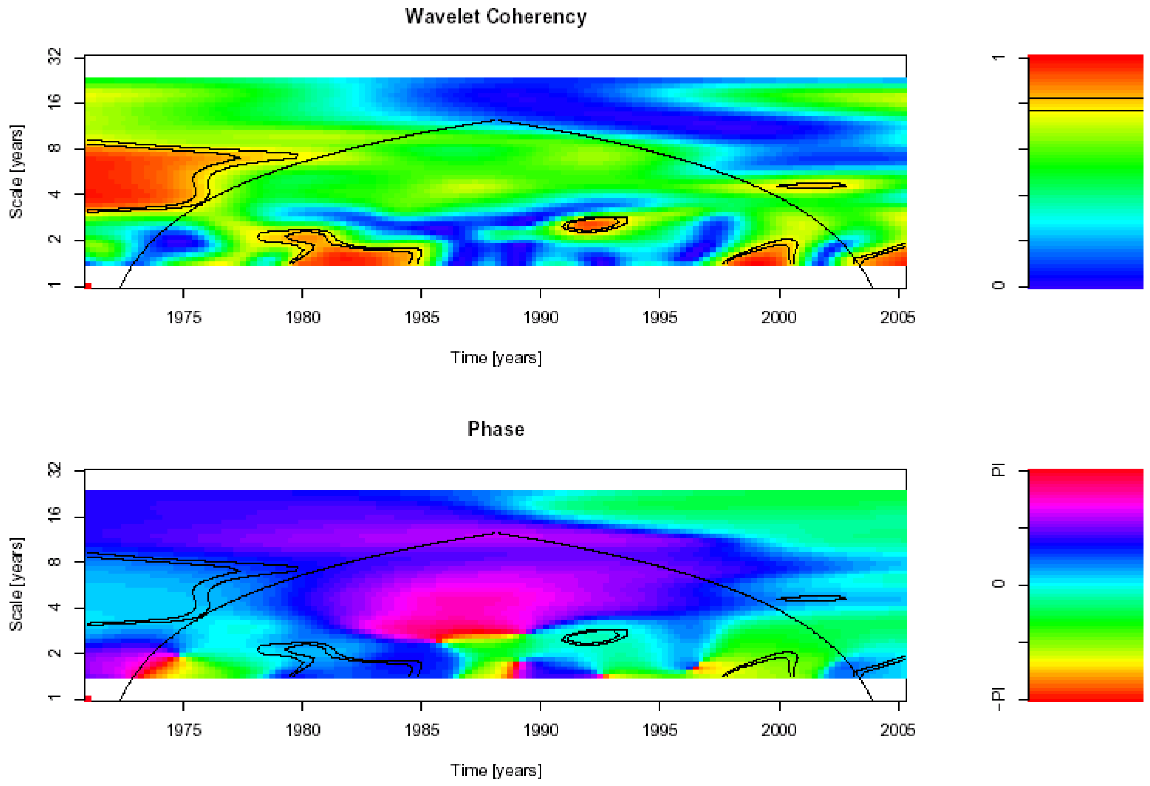


Figure 17: CWT for UK vs Euro area productivity

4 Spectral analysis using Hilbert Wavelet pairs (HWP)

4.1 Methodology

- Modified version of Nick Kingsbury's Dual-tree WT where data is operated on by two different filters - Craigmile and Whitcher (2004). Here the fiilters contain the same wavelet, except one of them is phased by a half sample delay from the other. So $A_0(f)$ and $B_0(f)$ form a Hilbert wavelet pair as long as $\theta(f) = \pi f$ in the formula

$$B_0(f) = A_0(f) \exp^{-i\theta(f)}$$

- 2 parameters required (Selesnick (2003)), denoted K and M, where K is the number of zero wavelet moments (which directly relates to the smoothness of the wavelet) and M represents the degree of approximation to the half sample delay (- as M increases this approximation improves) - K=4 and M=2 used here.

- MODHWT done by passing the 2 wavelet functions through the data and then let $\{(W_t^X, W_t^Y)^T : t \in \mathbb{Z}\}$ be the MODHWT detail crystals from two series X_t and Y_t.

- The time-varying cross spectrum of X_t and Y_t can then be defined as:

$$S_{XY}(\lambda_j, t) = E\left[W_{j,t}^X W_{j,t}^Y\right] \tag{8}$$

- Now from this spectral quantities can be calculated.

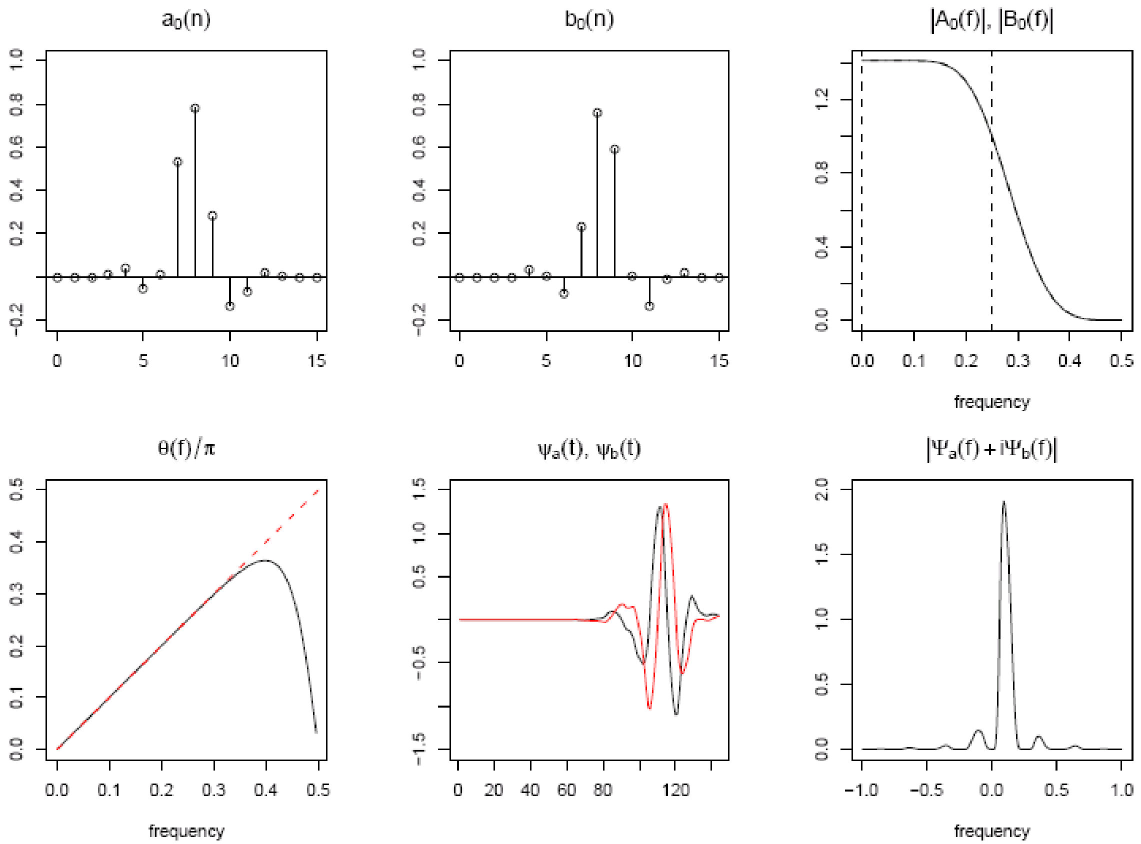
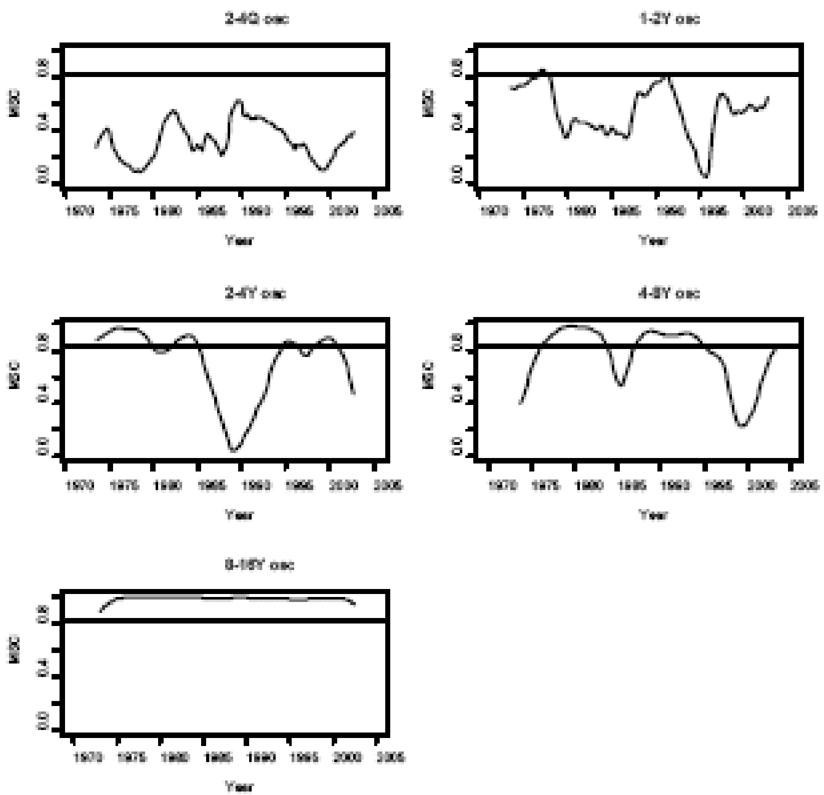
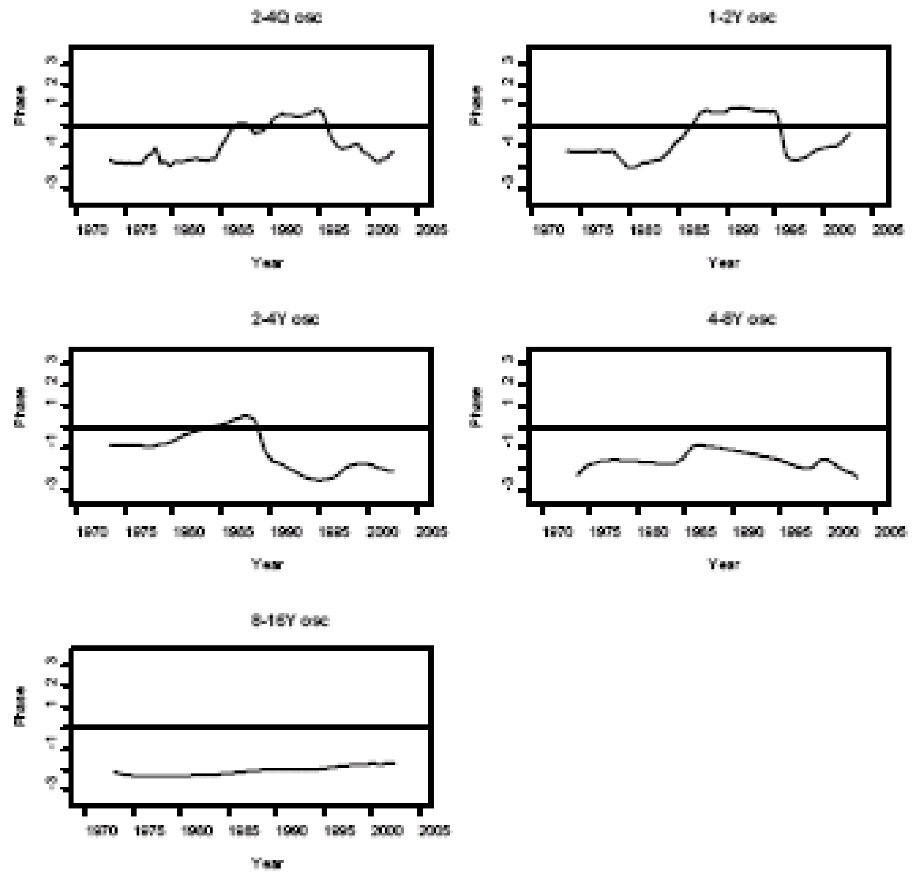


Figure 18: Source: Craigmile and Whitcher (2004)

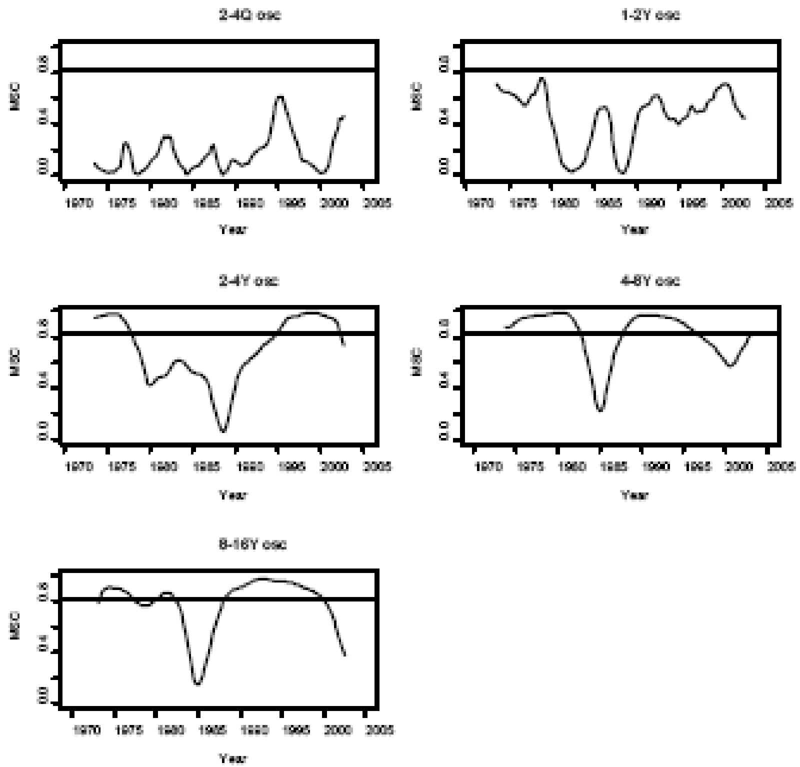
4.2 Results



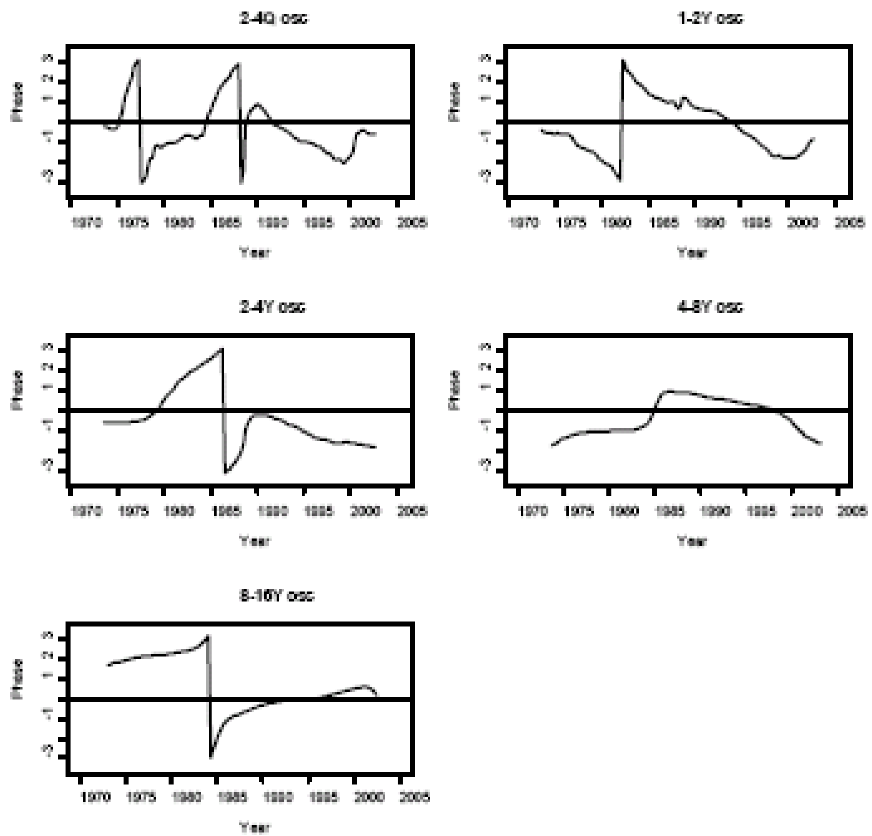
HWP Squared coherence plots for Euro area vs US productivity



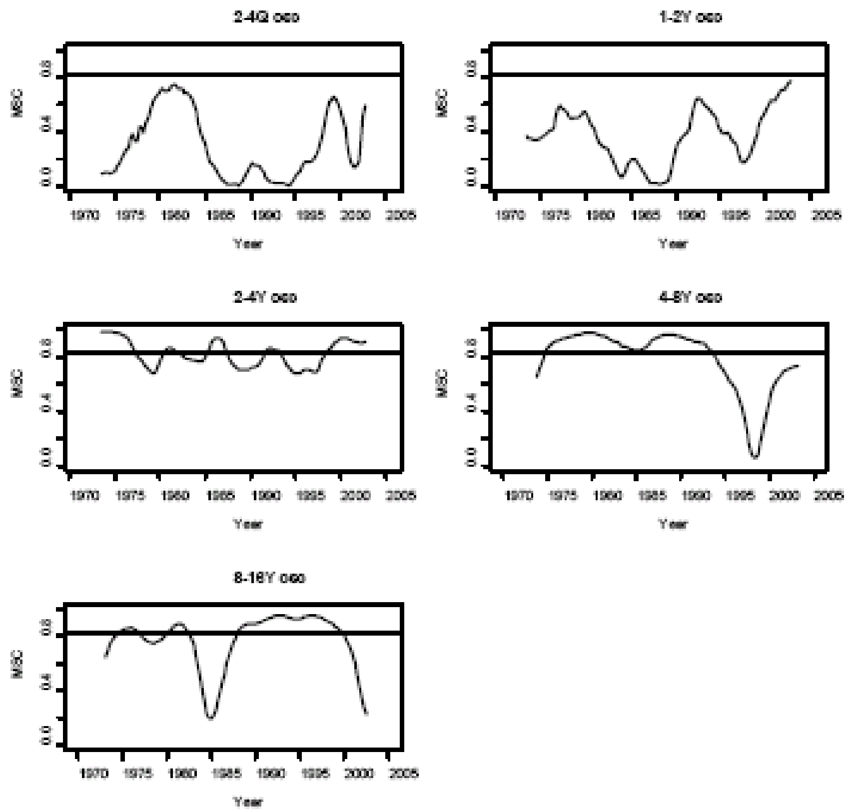
HWP phase plots for Euro area vs US productivity



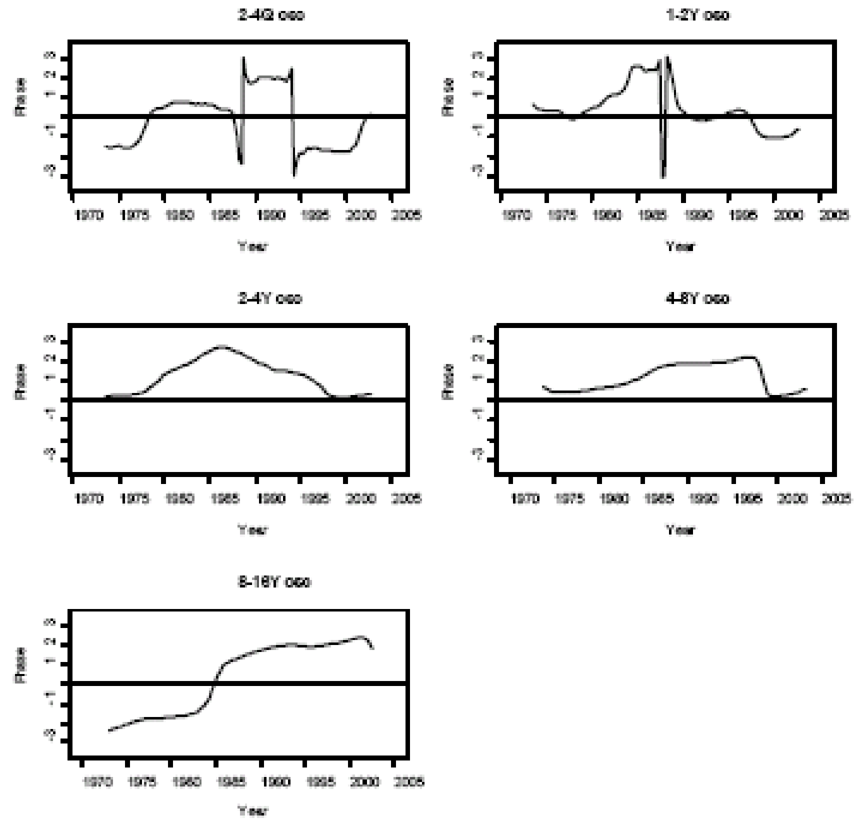
HWP Coherence plot for UK vs US productivity



HWP Phase plot for UK vs US productivity



HWP squared coherence plots for UK vs Euro area productivity



HWP phase plot for UK vs Euro area productivity

5 Discussion

i) productivity growth cycles below the 2 year frequency do not seem to be significant in terms of their coherence for any of the comparisons;

ii) the productivity cycles of the Euro area tend to lag behind those of the US at all cycles and the UK currently at longer cycles;

iii) there appears to have been a large dip in coherency during the mid-1980s, although this shows up at different frequencies depending on the comparison (for the Euro area vs the US it shows up in 2-8 year oscillations, for the UK vs the US in 2-16 year oscillations and for the UK vs the Euro area for 8-16 year oscillations only); and

iv) there appears to be a significantly coherent long cycle between the Euro area and the US, although detecting this cycle and its frequency is clearly difficult given the data limitations in this exercise.

6 Conclusions

a) cyclical behaviour in productivity growth is established as a stylized fact, and that while the frequency range differs by country or countries, the strongest component of this growth occurs at higher frequencies than that of the traditionally measured business cycle;

b) in the cases under investigation there is also likely a weak cycle which is at a much lower frequency than the business cycle also at work driving productivity growth;

c) over the period 1971-2005, while there are positive correlations for cycles near or at the business cycle frequency, none of the positive correlations between cycles for the Euro area, the US and the UK are significant

d) coherence at business cycle frequencies was high and significant in the 1970s, but during the 1980s there was a dip in this coherence, followed by some rebound, but not consistently; and

e) Euro area productivity cycles tend to lag those of the US, UK cycles lag only slightly against the US, and UK cycles tend to lead those of the Euro area. This results are not consistent across all cycle frequencies.