Short–Run Restrictions: An Identification Device?

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Motivation

- The econometrics of DSGE models has witnessed substantial advances over the recent years.
- Quite fashionable to bring DSGE models to the data.
  - Estimate deep parameters
  - Assess the relevance of propagation and amplification mechanisms
  - Conduct measures of fit

Motivation

- Variety of methods
  - Maximum Likelihood estimation (Altug, 1989, Ireland, 2004)
  - Generalized Method of Moments (Christiano and Eichenbaum, 1992, Burnside, Eichenbaum and Rebelo, 1993)
  - Bayesian techniques (Schorfheide, 2000, Smets and Wouters, 2003)
  - Minimum Distance Estimation (Rotemberg and Woodford, 1997, Christiano, Eichenbaum and Evans, 2005)
- This paper is concerned with the principles underlying the latest approach

Motivation

Minimum Distance Estimation

Estimate the structural parameters of DSGE models so as to minimize a weighted distance between

- IRF obtained from a Structural Vector AutoRegression ($\widehat{IRF}$) and
- theoretical IRF obtained from DSGE ($IRF(\theta)$).

$$\Longrightarrow \ \theta \in \mathrm{Argmin}_{\theta\in\Theta} \|IRF(\theta) - \widehat{IRF}\|_W$$

Motivation

3 main attractive features:

1. Bring structural and empirical approaches into closer conformity.
2. Set the focus only on those shocks that are relevant for the question under study
3. Not necessary to fully specify the whole stochastic structure of the DSGE model.

Motivation

- Key: Need to pin down the structural shocks, prior to estimating the DSGE parameters.
- Identification Restrictions to get IRFs
  - Long run restrictions (Blanchard and Quah (1989) and Galí (1999)) (close to most models)
  - Short–run Restrictions (Sims, 1980, Christiano, Eichenbaum, and Evans, 2005)

Motivation

Christiano, Eichenbaum and Vigfusson (2005):

1. SR performs better than LR restrictions.
2. SR restrictions correctly and precisely pin down the shocks.

Conjecture

SVARs with SR restrictions are a useful assessment device when constructing and evaluating a DSGE model.

Motivation

Key to this method:

DSGE and SVAR share the same restrictions (SR and/or LR)

- With SR, the DSGE and the VAR should share the same recursive structure
- Imposes some information restrictions on the observation of shocks
- Common intuition: This should affect the estimation

Motivation

- Empirically: This choice seems to be **innocuous!**
- Ex1: CEE (2005): Information sets restrictions have very little impact on the dynamic properties of their model in response to a money shock. (True in other monetary models)
- Ex2: CEV (2005): The response of hours to a permanent technology shock is left unaffected by observability restrictions.
- Very nice property!

This paper

- Is this always true? (Build an example to understand)
- When are parameters invariant to the short–run identification scheme used to identify shocks?
- Show how SR restrictions can be used to check identification

(A Tentative) Plan of the Talk

- The Minimum Distance Estimation with SR restrictions
- A fully fledged DSGE Model
- Estimate the model
- Assess the Role of Timing Restrictions
- Using timing restrictions to reveal identification problems
- Conclude

MDE with SR Restrictions

IRFs from the VAR

- Consider a sequence of data $\{x_t\}_{t=1}^{T}$, $x_t(n_x \times 1)$
- VAR representation

$$A(L)x_t = \varepsilon_t$$

where $A(L) = (I - A_1 L - \ldots - A_\ell L^\ell)$ and $\mathscr{E}\varepsilon_t = 0$ and $\mathscr{E}\{\varepsilon_t \varepsilon_t'\} = \Sigma$.

- Structural Shocks: $\varepsilon_t = S\eta_t$,
- VMA(∞) representation

$$x_t = C(L)\eta_t$$

where $C(L) = \sum_{i=0}^{\infty} C_i L^i = A(L)^{-1} S$.

MDE with SR Restrictions

IRFs from the VAR

- Short–run Restrictions:

$$S = C(0) = \begin{pmatrix} \times & 0 \\ \times & \times \end{pmatrix} \Longrightarrow S = \text{chol}(\Sigma)$$

- Response of variable $x_{i,t}$ to shock j at horizon h:

$$I^{ij}(h) \equiv \frac{\partial x_{i,t+h}}{\partial \eta_{j,t}} = C_h^{(i,j)}$$

- $\widehat{I}_T(\mathcal{H})$ denotes the vector collecting the IRFs from the SVAR for the horizons $\mathcal{H} = \{1, \ldots, h\}$.

MDE with SR Restrictions

IRFs from the DSGE

- Generic linear(ized) DSGE model

$$\mathscr{E}_t^{\star}\left\{\sum_{i=-\tau_b}^{\tau_f} H_i\left(\theta_1\right) y_{t+i} + \sum_{i=-r_b}^{r_f} R_i\left(\theta_1\right) s_{t+i}\right\} = 0$$

θ_1: vector of deep parameters, y_t: endogenous variables, s_t: forcing variables

- Expectations:

$$\mathscr{E}_t^{\star}(F(y_i, s_j)\} = \begin{bmatrix} \mathscr{E}\{F_1(y_i, s_j)|\mathscr{I}_{1,t}\} \\ \vdots \\ \mathscr{E}\{F_{n_y}(y_i, s_j)|\mathscr{I}_{n,t}\} \end{bmatrix}$$

where we place restrictions

MDE with SR Restrictions

IRFs from the DSGE

- Exogenous forcing variables

$$s_t = P(\theta_2) s_{t-1} + Q(\theta_2) \varepsilon_t,$$

  θ_2 is a vector of parameters governing the dynamics of the forcing variables.

- Solution of the DSGE

$$y_t = D_1(\theta_1) y_{t-1} + \cdots + D_{\tau_b}(\theta_1) y_{t-\tau_b} + M(\theta_1, \theta_2) s_t.$$

- IRFs: $I(\theta, \mathcal{H})$, where $\theta \equiv (\theta_1, \theta_2)$.

MDE with SR Restrictions

MDE Estimator

The MDE estimator of the structural parameters of the DSGE is

$$\widehat{\theta}_T = \underset{\theta\in\Theta}{\operatorname{argmin}} \; (I(\theta,\mathcal{H}) - \widehat{I}_T(\mathcal{H}))' W_T (I(\theta,\mathcal{H}) - \widehat{I}_T(\mathcal{H})),$$

where W_T denotes the inverse of a consistent estimate of the covariance matrix of $\widehat{I}_T(\mathcal{H})$

- Overidentification test (J–test)
- Distribution of $\widehat{\theta}_T$: $\widehat{\theta}_T \rightsquigarrow \mathcal{N}\left(0, V(\widehat{\theta}_T)\right)$ with

$$V(\widehat{\theta}_T) = \left(\left.\frac{\partial I(\theta,\mathcal{H})'}{\partial\theta}\right|_{\theta=\widehat{\theta}_T} W_T \left.\frac{\partial I(\theta,\mathcal{H})}{\partial\theta'}\right|_{\theta=\widehat{\theta}_T} \right)^{-1}.$$

Where are we?

- The Minimum Distance Estimation with SR restrictions
- **A fully fledged DSGE Model**
- Estimate the model
- Assess the Role of Timing Restrictions
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- Conclude

A DSGE Model

- Account for output–hours dynamics
- Want an empirically relevant model
- Needs propagation mechanisms to allow for a (fair) formal test of the model.
- Introduce real frictions

A DSGE Model

The Household

- Standard!
- intertemporal expected utility function of the representative household

$$\mathscr{E}\left[\sum_{s=0}^{\infty}\beta^{s}\left\{\log\left(C_{t+s}-bC_{t+s-1}\right)-\chi N_{t+s}\right\}\middle|\mathscr{I}_{t}\right]$$

A DSGE Model

Technology

- Standard Cobb–Douglas!

$$Y_t = K_t^{\alpha} (Z_t N_t)^{1-\alpha},$$

- Forcing variable

$$\log(Z_t) = \gamma_z + \log(Z_{t-1}) + \sigma_z \varepsilon_{z,t},$$

- Capital formation

$$K_{t+1} = (1-\delta) K_t + \left[1 - \mathcal{S}\left(\frac{I_t}{I_{t-1}}\right)\right] I_t$$

A DSGE Model

2 versions of the Model

1. $\mathcal{M}_1$: **All** decisions are taken **after** the realization of the technology shock at t ($\mathscr{I}_t = \{Z_{t-i}; i = 0, \ldots, \infty\}$).
2. $\mathcal{M}_2$: Follow CEV and assume that
    1. Labor supply decisions are taken **prior to** observing the technology shock ($\mathscr{I}_t = \{Z_{t-i}; i = 1, \ldots, \infty\}$).
    2. Consumption and investment are however chosen **after** the shock is observed.

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Setting some parameters

- set $\{\alpha, \beta, \delta, \gamma_z\}$ to standard values

Baseline parametrization

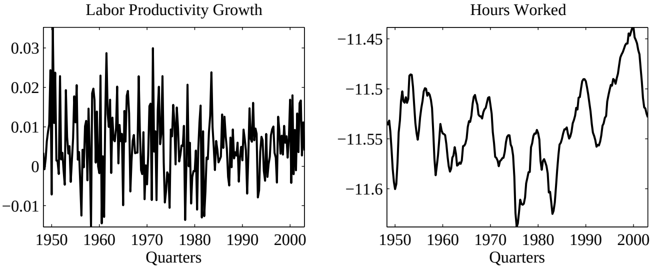
γ_z	α	δ	β
1.005	0.360	0.025	0.990

- Estimate parameters pertaining to dynamics (persistence, impact):

  b (HP), ξ (IAC) ,σ_z (Std of Shock).

Which VAR?

- Run a VAR on growth rate of labor productivity in the nonfarm business–sector, and hours of all persons



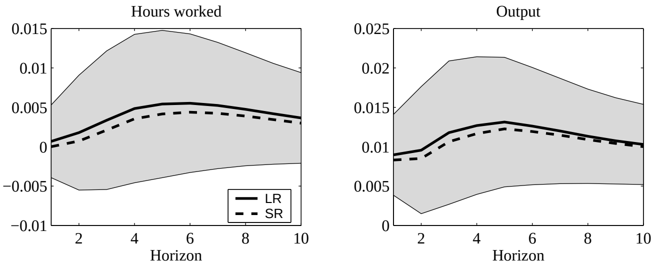
- 4 lags, Hours in levels (CEV)

Which IRF?

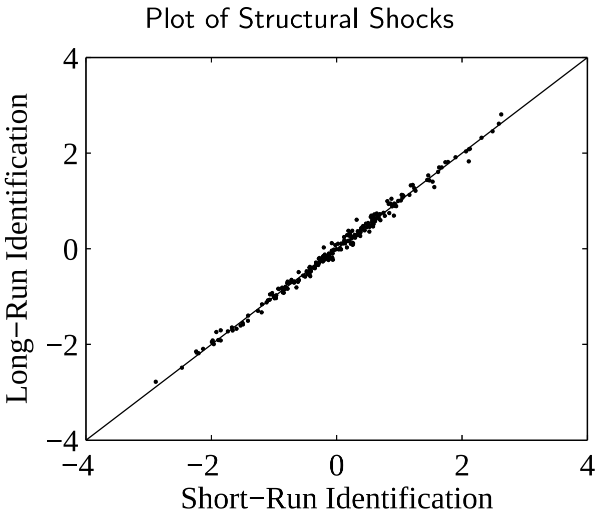
- Dynamics following the technology shock
- Standard to use LR restriction to get technology shocks
- We will use a short–run restriction!
- For this sample: observationally equivalent!!

Which IRF?

Impulse Response to a Technology Shock (Actual Data)



Which IRF?


Plot of Structural Shocks
Long−Run Identification
Short−Run Identification
4
2
0
−2
−4
−4
−2
0
2
4

Estimation

- To be matched: IRFs of y and h to the technology shock
- DSGE: model with information lags (model $\mathcal{M}_2$)
- 2 experiments
  1. RBC experiment: $\theta = (\sigma_z)$ and set $b = \xi = 0$
  2. Full model experiment: $\theta = (b, \xi, \sigma_z)$

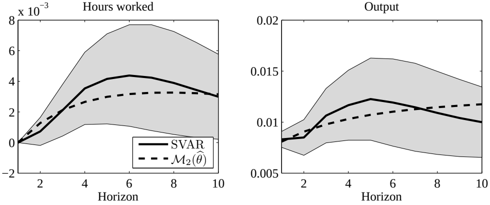
Estimation Results

	b	ξ	σ_z	J–stat
RBC Model	–	–	0.0102	102.6938
			(0.0004)	[0.00]
Full Model	0.7253	3.3942	0.0126	9.2817
	(0.4337)	(1.7215)	(0.0006)	[95.30]

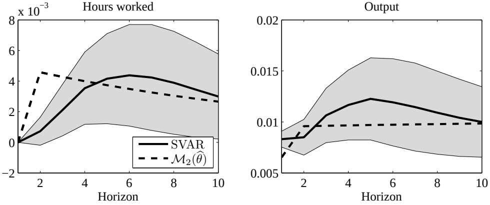
Note: Standard deviation into parenthesis, p–values (%) into brackets.

Fitting Properties

(a) Full Model



(b) RBC Model



- What have we done so far?
- Not much!
- But we have a model that provides a satisfactory representation of the data!
- The model can be taken as a good approximation of the DGP of the data
- Can work with it

Where are we?

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- **Assess the Role of Timing Restrictions**
- Using timing restrictions to reveal identification problems
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Assessing the Role of Timing Restrictions

- Follow CEE (2005): Should the restrictions be innocuous, they should not alter the dynamics of the model!
- Compare IRFs from the unrestricted ($\mathcal{M}_1(\widehat{\theta}_T)$) and the restricted ($\mathcal{M}_2(\widehat{\theta}_T)$) versions of the IRFs

Assessing the Role of Timing Restrictions

Case 1: $\mathcal{M}_1(\widehat{\theta}_T) = \mathcal{M}_2(\widehat{\theta}_T)$

- Confirms CEE, CEV and other findings
- The method delivers the same estimated values of parameters for the 2 models
- Does not select a theory, just a way of identifying parameters.
- MDE treats fundamental parameters as deep parameters.

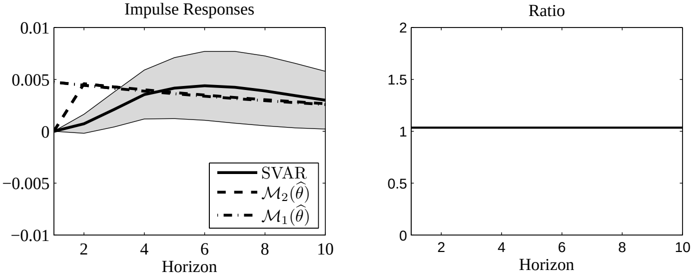
Assessing the Role of Timing Restrictions

Case 2: $\mathcal{M}_1(\widehat{\theta}_T) \neq \mathcal{M}_2(\widehat{\theta}_T)$

- SR restrictions have long–lasting effects
- Far from being innocuous: Identifies a theory
- Good news: SR are very informative
- Refute previous findings Estimation is sensitive to the decomposition.
- Need to be cautious

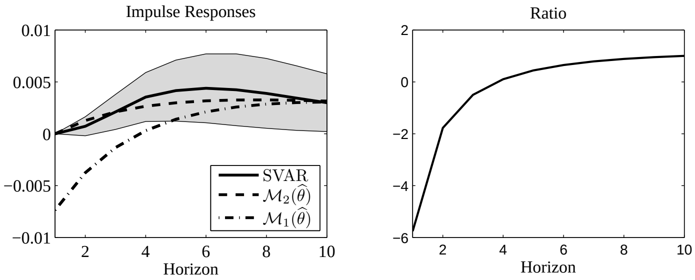
Assessing the Role of Timing Restrictions

The RBC version



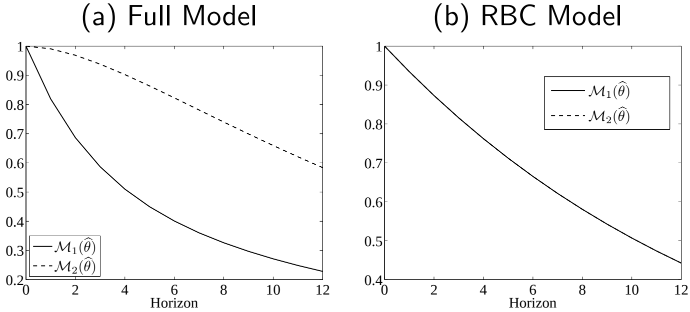
Assessing the Role of Timing Restrictions

The full version



Assessing the Role of Timing Restrictions

- Stems from the size of the propagation mechanisms in each version
- The full version propagates SR restrictions!
- This is confirmed by the autocorrelation function of the two models

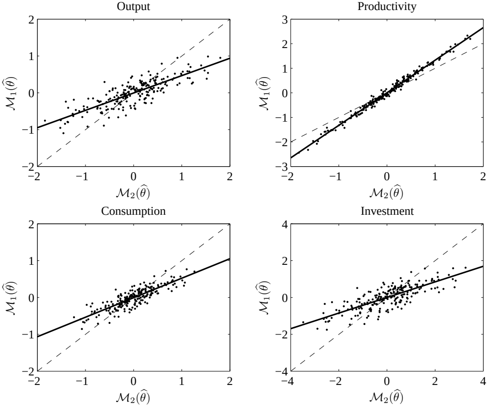


Assessing the Role of Timing Restrictions

- Feed each model with the technology shocks identified by the VAR.
- Plot the series obtained from $\mathcal{M}_1(\widehat{\theta}_T)$ against $\mathcal{M}_2(\widehat{\theta}_T)$
- Illustrate how observationally equivalent (different) the two restriction sets are

Assessing the Role of Timing Restrictions

$\mathcal{M}_1(\widehat{\theta}_T)$ versus $\mathcal{M}_2(\widehat{\theta}_T)$



Assessing the Role of Timing Restrictions

- Restricting the ability of agents to react to shocks actually puts a lot of structure on the data.
- Far from being innocuous.
- Provided the model possesses internal mechanisms.
- Imposing short–run restrictions on the VAR model amounts to selecting a theoretical model.
- Structure of information may matter for estimation.
- Is it useful?

Where are we?

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- Conclude

Counterfactuals

- Aim: Use SR restrictions to reveal identification problems
- Ask a very simple question:

Is it possible to replicate $\mathcal{M}_2$ with $\mathcal{M}_1$?

- Otherwise stated: Does there exist $\widetilde{\theta}_T$ such that $\mathcal{M}_1(\widetilde{\theta}_T) = \mathcal{M}_2(\widehat{\theta}_T)$?
- If yes, do we have $\widetilde{\theta}_T = \widehat{\theta}_T$?

Counterfactuals

- Does there exist $\widetilde{\theta}_T$ such that $\mathcal{M}_1(\widetilde{\theta}_T) = \mathcal{M}_2(\widehat{\theta}_T)$?
- Find $\widetilde{\theta}_T$ so as to solve

$$\underset{\theta \in \Theta}{\operatorname{argmin}} \left(I_1(\theta, \mathcal{H}) - I_2(\widehat{\theta}_T, \mathcal{H}) \right) \Omega_2(\widehat{\theta}_T) \left(I_1(\theta, \mathcal{H}) - I_2(\widehat{\theta}_T, \mathcal{H}) \right)'$$

$I_j(\theta, \mathcal{H})$ are IRFS for model j, and

$$\Omega_2(\widehat{\theta}_T) = \left(\left. \frac{\partial I_2(\theta, \mathcal{H})'}{\partial \theta} \right|_{\theta = \widehat{\theta}_T} V(\widehat{\theta}_T) \left. \frac{\partial I_2(\theta, \mathcal{H})}{\partial \theta'} \right|_{\theta = \widehat{\theta}_T} \right)^{-1}.$$

- Over–identification test for the equivalence between models (with $nh - \dim(\theta)$ d.f.)

Counterfactuals

- If no then SR restriction really matter,
- If yes: do we have $\widetilde{\theta}_T = \widehat{\theta}_T$?
- Can be tested by

$$\mathcal{W}_1(\widetilde{\theta}_T, \widehat{\theta}_T) = (\widetilde{\theta}_T - \widehat{\theta}_T)' V(\widehat{\theta}_T)^{-1} (\widetilde{\theta}_T - \widehat{\theta}_T)$$

  which is χ^2 with $\dim(\theta)$ d.f.
- If yes, SR restrictions do not matter (RBC model)
- If no, use it further.

Counterfactuals

	ξ	b	σ_z	J–stat	$\mathcal{W}_1(\widetilde{\theta}_T, \widehat{\theta}_T)$
Benchmark	3.3942	0.7253	0.0126		
Output	0.9750	-0.0404	0.0124	0.1479	6.8710
	(0.3996)	(0.0929)	(3.4541e-4)	[100.00]	[7.60]
Productivity	1.0566	-0.1101	0.0130	0.2373	8.0250
	(0.6757)	(0.1778)	(3.6125e-4)	[99.99]	[4.50]
Consumption	1.3330	0.4950	0.0128	0.04392	2.9478
	(3.5999)	(0.5748)	(9.0620e-4)	[100.00]	[39.90]
Investment	0.2875	0.9855	0.0225	0.0677	616.5846
	(0.9353)	(0.0540)	(0.0422)	[100.00]	[0.00]
All	1.4061	-0.0414	0.0127	5.73815	5.7266
	(0.2972)	(0.0770)	(1.4832e-4)	[100.00]	[12.60]

Note: Standard deviation into parenthesis, p–values (%) into brackets.

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Counterfactuals

Going further

- Assume $\widetilde{\theta}_T \neq \widehat{\theta}_T$
- Feed model $\mathcal{M}_2$ with $\widetilde{\theta}_T$
- Is the restricted model $\mathcal{M}_2$ affected by this change of parameters?
- Formal test

$$\mathcal{W}_2(\widetilde{\theta}_T, \widehat{\theta}_T) = \left(I_2(\widetilde{\theta}_T, \mathcal{H}) - I_2(\widehat{\theta}_T, \mathcal{H})\right) \Omega_2(\widehat{\theta}_T) \left(I_2(\widetilde{\theta}_T, \mathcal{H}) - I_2(\widehat{\theta}_T, \mathcal{H})\right)'$$

- If one cannot reject the null: Severe identification problem!

Counterfactuals

Going further

- Applying the test

$$\mathcal{W}_2(\widetilde{\theta}_T, \widehat{\theta}_T)$$

Output	Productivity	Consumption	Investment
1.5570	1.6873	0.1736	56.9746
[99.67]	[99.55]	[99.99]	[0.00]

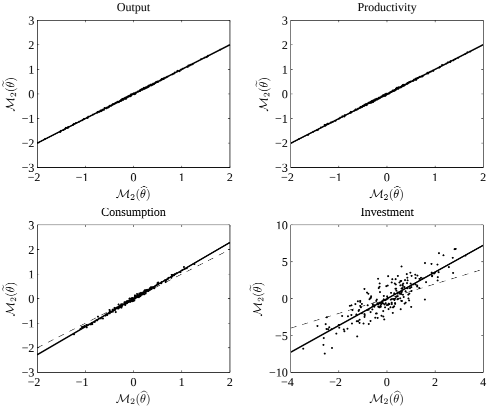
Note: p–values (%) into brackets.

- Very bad news!
- $\mathcal{M}_2(\widetilde{\theta}_T)$ and $\mathcal{M}_2(\widehat{\theta}_T)$ share the same dynamic properties.
- Identification problem!

Counterfactuals

Going further

$\mathcal{M}_2(\widehat{\theta}_T)$ versus $\mathcal{M}_2(\widetilde{\theta}_T)$



Concluding Remarks

- This paper investigates the quantitative implications of restricting the information sets conditional on which decisions are taken in DSGE models.
- Propose a variety of simple statistical tools designed to assess the role of timing restrictions on aggregate dynamics.
- Use them on a fully–fledged DSGE model which we formally take to US data.

Concluding Remarks

- Our results indicate that restricting the information set is of
  - No substantial consequences if one is dealing with a version of our DSGE model with weak internal propagation mechanisms
  - Big consequences when the model possesses strong propagation mechanisms
- Illustrates how to use these restrictions to reveal identification problems.