Prior Choice and DSGE Model Comparisons

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Estimation and Empirical Validation of Structural Models
EACBN/CEPR Swiss National Bank, August 29, 2006

Motivation

- The new generation of new-Keynesian DSGE models (Christiano et al., Smets and Wouters ...) fits the data reasonably well, and hence can be used for policy analysis at Central Banks.

- These models contain many bells and whistles (and persistent shocks) – some are more "structural" than others.

- Which features are really needed, and which can we get rid of?

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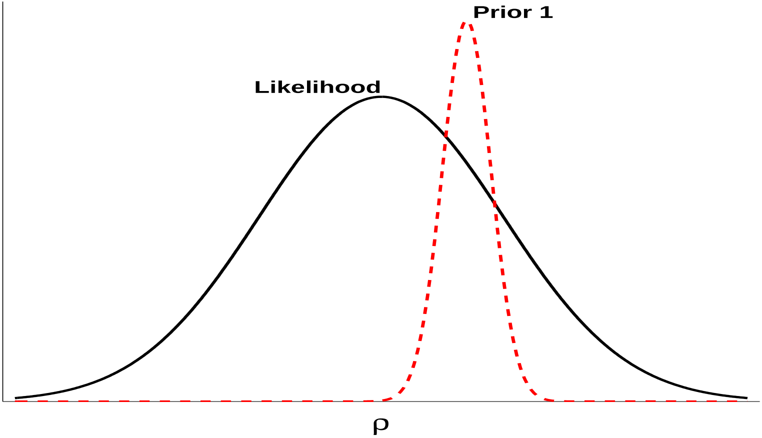
- These models contain many bells and whistles (and persistent shocks) – some are more "structural" than others.
- Which features are really needed, and which can we get rid of?
- Two approaches for model comparison:
  - Impulse responses (CEE)
  - Bayesian model comparisons via Marginal Likelihoods (Smets and Wouters)

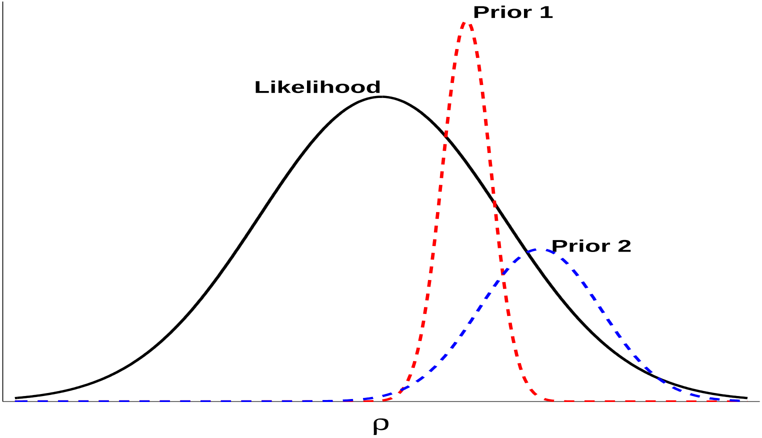
Priors and Model Comparisons

- The marginal likelihood is the integral of the likelihood with respect to the prior



Priors and Model Comparisons

- The marginal likelihood is the integral of the likelihood with respect to the prior
- . . . hence the choice of the prior matters



Motivation - continued

DSGE model parameters can be divided into two sets:

1. **Deep** parameters – for which we can have priors based on micro evidence, economic theory . . .

2. **Auxiliary** parameters – correlation and standard deviations of exogenous shocks.
   - Hard to have intrinsic beliefs about the driving process of these unobservable shocks
   - Hence researchers have informally chosen these priors to match moments of the endogenous variables . . . or simply taken these priors from other papers . . .
   - . . . and used the same prior across different models.

Objective of the paper

Build a procedure to form priors for the auxiliary parameters such that these priors reflect our beliefs on the moments of the endogenous variables.

- Introduce dependence among parameters.
- In Bayesian model comparisons, make sure priors do not unduly penalize some specifications.

Identifying Backward Looking Behaviour in a Simple Example

- Take two models:

$$\mathcal{M}_1: \quad y_t = \frac{1}{\alpha} \boldsymbol{E}_t[y_{t+1}] + \rho_1 y_{t-1} + u_t, \quad u_t = \epsilon_t \sim iid(0, \sigma^2).$$

$$\mathcal{M}_2: \quad y_t = \frac{1}{\alpha} \boldsymbol{E}_t[y_{t+1}] + u_t, \quad u_t = \rho_2 u_{t-1} + \epsilon_t \sim iid(0, \sigma^2).$$

- Solution:

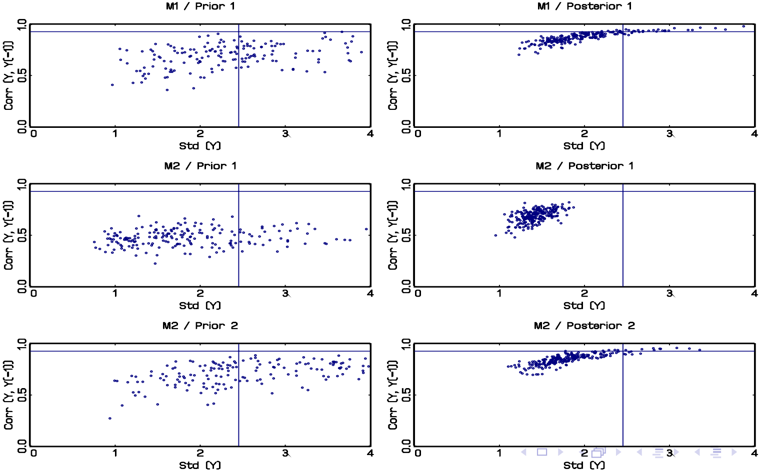
$$\mathcal{M}_1: \quad y_t = \frac{1}{2}(\alpha - \sqrt{\alpha^2 - 4\rho_1\alpha}) y_{t-1} + \frac{2\alpha}{\alpha + \sqrt{\alpha^2 - 4\rho_1\alpha}} \epsilon_t,$$

$$\mathcal{M}_2: \quad y_t = \rho_2 y_{t-1} + \frac{1}{1 - \rho_2/\alpha} \epsilon_t$$

- Lubik and Schorfheide, Bayer and Farmer.

Priors and Model Comparisons in the Simple Example

1. Prior 1: Use **same prior** for $\mathcal{M}_1$ and $\mathcal{M}_2$
2. Prior 2: Choose prior for and $\mathcal{M}_2$ so that the **same** "a priori" **implications for moments** of the endogenous variables.



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Specification	$\ln p(Y)$
Model $\mathcal{M}_1$, Prior 1	-105.93
Model $\mathcal{M}_2$, Prior 1	-123.53
Model $\mathcal{M}_2$, Prior 2	-105.70

DSGE Model

- Model is a variant of Altig, Christiano, Eichenbaum, and Linde (2002); Christiano, Eichenbaum, and Evans (2004), Smets and Wouters (2003).

- Continuum of households, they maximize:

$$\boldsymbol{E}_t \sum_{s=0}^{\infty} \beta^s [\log(C_{t+s} - hC_{t+s-1}) - \frac{\varphi_{t+s}}{1+\nu_l} L_{t+s}^{1+\nu_l} \dots$$
$$\dots + \frac{\chi}{1-\nu_m} \left(\frac{M_{t+s}}{Z_{t+s} P_{t+s}} \right)^{1-\nu_m}],$$

- Accumulate capital: $\bar{K}_t = (1-\delta)\bar{K}_{t-1} + \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t,$

- Rent out "effective" capital $K_t = u_t \bar{K}_{t-1}$ and pay the utilization cost $a(u_t)\bar{K}_{t-1}$.

DSGE Model – continued

- Sticky wages: reset wages with probability $1-\zeta_w$.

- Partial indexation: $W_{t+s} = \left(\Pi_{l=1}^{s}(\pi_* e^{\gamma})^{1-\iota_w}(\pi_{t+l-1}e^{\gamma})^{\iota_w}\right)\tilde{W}_t$.

- Continuum of intermediate goods producers, who use Cobb-Douglas technology:
$$Y_t(i) = K_t(i)^{\alpha}(Z_t L_t(i))^{1-\alpha}$$
with unit root in technology: $z_t = \log(Z_t/Z_{t-1})$ has mean γ.

- Sticky prices: reset prices with probability $1-\zeta_p$ + Partial indexation (ι_p).

- $Y_t(i)$ packed into a composite good: $Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_{f,t}}} di\right]^{1+\lambda_{f,t}}$.

DSGE Model – continued

- Government balances budget

$$P_t G_t + R_{t-1} B_{t-1} + M_{t-1} = T_t + M_t + B_t$$

where $G_t = (1 - 1/g_t) Y_t$.

- The central bank follows a nominal interest rate rule:

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*} \right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi_*} \right)^{\psi_1} \left(\frac{Y_t}{Y_t^*} \right)^{\psi_2} \right]^{1-\rho_R} \sigma_R e^{\epsilon_{R,t}}$$

where Y_t^* is the stochastic steady state level of output.

- All shocks follow an AR(1) process (except $\epsilon_{R,t}$, which is iid).

Measurement equations

- Output growth (log differences, quarter-to-quarter, in %):
  $100 \times (\ln Y_t - \ln Y_{t-1}) = 100 \times (\hat{y}_t - \hat{y}_{t-1} + \hat{z}_t) + 100\gamma$
- Hours worked (log, in %): $100 \times \ln L_t = 100 \times \hat{L}_t + \ln L^{adj}$
- Labor Share (log, in %):
  $100 \times \ln LS_t = 100 \times (\hat{w}_t + \hat{L}_t - \hat{y}_t + \ln(1-\alpha))$
- Inflation (annualized, in %):
  $400 \times (\ln P_t - \ln P_{t-1}) = 400\hat{\pi}_t + 400 \ln \pi^*$
- Nominal interest rate (annualized, in %):
  $400 \times (\ln R_t) = 4 \times 100\hat{R}_t + 400 * \ln R^*$
- 100 quarters of data ending QIV-2005.

Dummy Observation Prior

1. Split θ into $\theta = [\theta_1\ \theta_2]$ where θ_1 collects the "deep" parameters (prior distributions based on micro evidence) and θ_2 is a sub-vector of auxiliary parameters.

2. Express your beliefs for the moments of the endogenous variables in terms of the matrices: Γ^*_{YY} , $\Gamma^*_{YY_{-1}}$, $\Gamma^*_{Y_{-1}Y_{-1}}$.

Dummy Observation Prior ...

3. Construct the VAR(1) approximation of the DSGE model: $\Phi_*(\theta),\ \Sigma_*(\theta)$.

4. Define the quasi-likelihood of the DSGE model as:

$$\mathcal{L}(\theta|\Gamma^*_{YY}, \Gamma^*_{Y_{-1}Y}, \Gamma^*_{Y_{-1}Y_{-1}}) = |\Sigma_*(\theta)|^{-(T^*+n+1)/2}$$
$$\times \exp\left\{ -\frac{T^*}{2} tr\left[\Sigma_*(\theta)^{-1} (\Gamma^*_{YY} - 2\Phi_*(\theta)\Gamma^*_{Y_{-1}Y} + \Phi'_*(\theta)\Gamma^*_{Y_{-1}Y_{-1}}\Phi_*(\theta) \right] \right\}.$$

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5. Rather than the standard prior $\pi_S(\theta_2)$ for the θ_2 parameters:

$$p(\theta_1,\theta_2) = \pi(\theta_1)\pi_S(\theta_2).$$

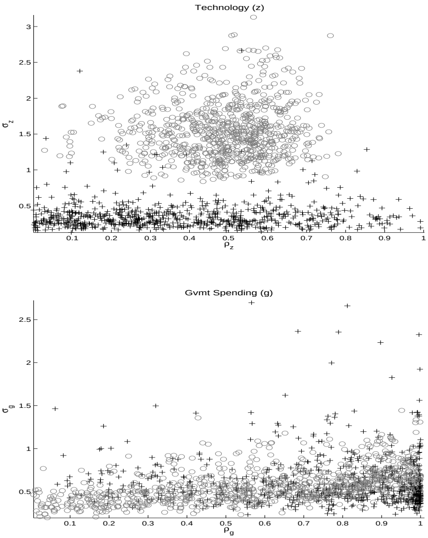
generate the prior as the product of an “uninformative” prior $\pi_U(\theta_2)$ and the quasi-likelihood:

$$p_*(\theta_1,\theta_2) = \pi(\theta_1)\ \ c_1(.)\mathcal{L}(\underline{\theta}_1,\theta_2|\Gamma^*)\pi_U(\theta_2).$$

where $c_1(.)$ guarantees the prior integrates to one.

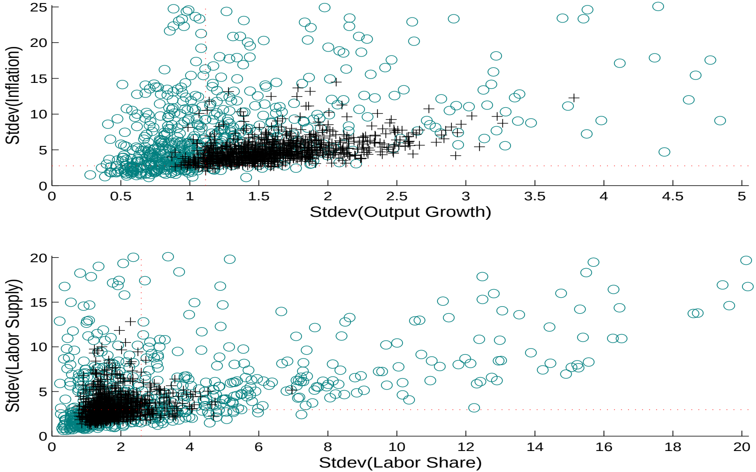
Dependence

Correlation among the Shock Parameters



Dependence

...and implications for the variance of the endogenous variables



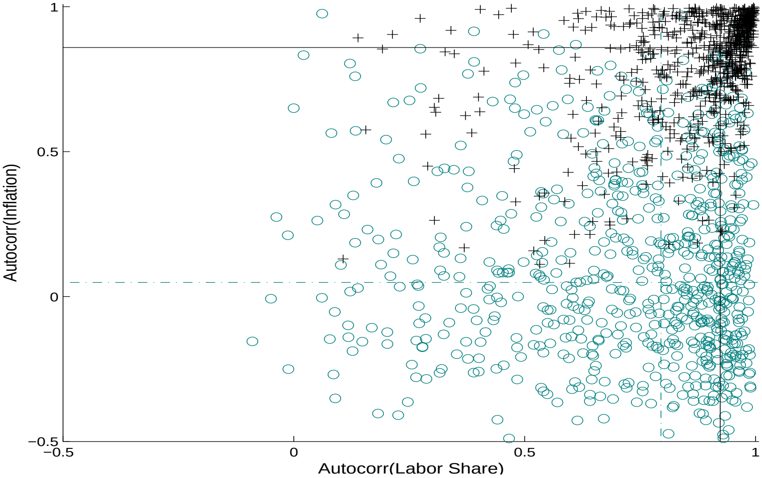
Empirical Analysis

Questions:

- Do we need sticky wages/sticky prices, and why?
- Do we need indexation in the Phillips curve?

Assessing the Importance of Nominal Rigidities

Standard Prior, Flex Wages and Prices vs Benchmark



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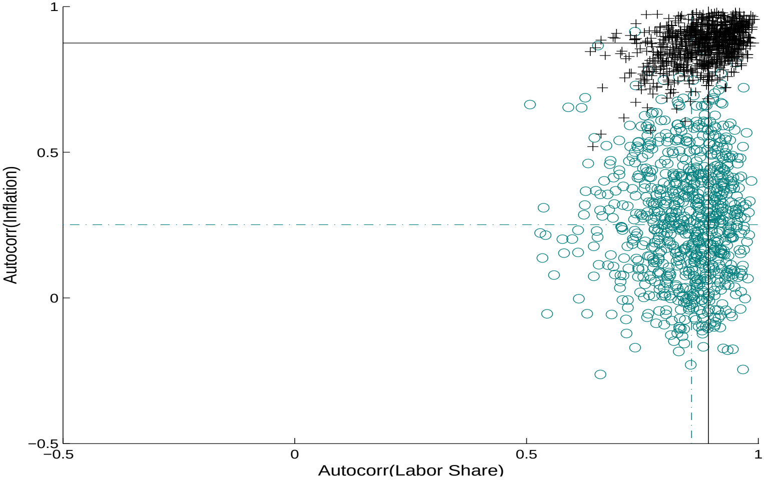
Dummy Obs. Prior, Flex Wages and Prices vs Benchmark



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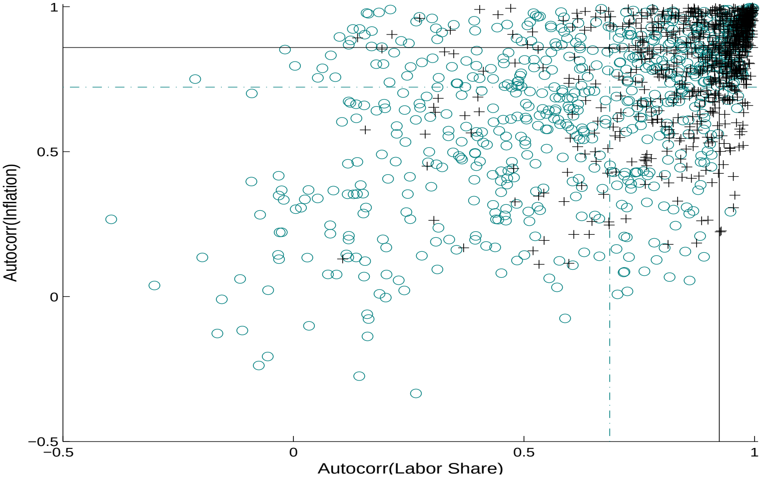

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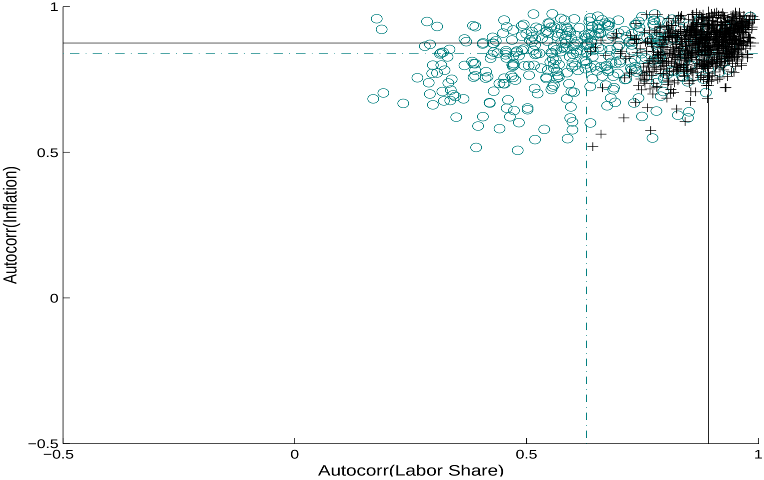
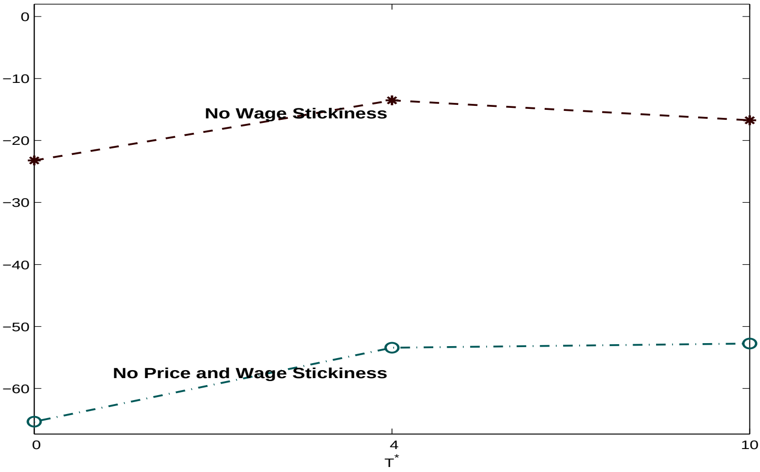


Priors for Auxiliary Parameters

	Standard Prior		Dummy Obs. Prior Baseline		Dummy Obs. Prior Flex. Wages		Dummy Obs. Prior Flex. Wages, Prices	
	Mean	StdDev	Mean	StdDev	Mean	StdDev	Mean	StdDev
ρ_z	0.400	0.250	0.489	0.129	0.332	0.118	0.326	0.122
ρ_ϕ	0.750	0.250	0.692	0.194	0.769	0.199	0.586	0.338
ρ_{λ_f}	0.750	0.250	0.843	0.120	0.799	0.146	0.884	0.067
ρ_g	0.750	0.250	0.597	0.278	0.840	0.204	0.922	0.141
σ_z	0.376	0.194	1.549	0.388	1.613	0.371	1.667	0.405
σ_ϕ	3.755	1.955	5.392	2.646	1.901	0.783	1.832	0.918
σ_{λ_f}	0.376	0.194	0.191	0.086	0.230	0.084	0.732	0.172
σ_g	0.626	0.323	0.577	0.204	0.789	0.406	0.822	0.320
σ_r	0.250	0.130	0.398	0.115	0.410	0.101	0.414	0.132

Marginal Likelihood Relative to Benchmark



Indexation and the Phillips Curve

$$\begin{aligned}\widehat{\pi}_t &= \frac{(1-\zeta_p\beta)(1-\zeta_p)}{(1+\iota_p\beta)\zeta_p}\left[\widehat{mc}_t + \frac{\lambda_f}{1+\lambda_f}\widehat{\lambda}_{f,t}\right] \\ &\quad + \frac{\iota_p}{1+\iota_p\beta}\widehat{\pi}_{t-1} + \frac{\beta}{1+\iota_p\beta}\boldsymbol{E}_t[\widehat{\pi}_{t+1}].\end{aligned}$$

Questions:

1. Is there indexation?

Indexation and the Phillips Curve

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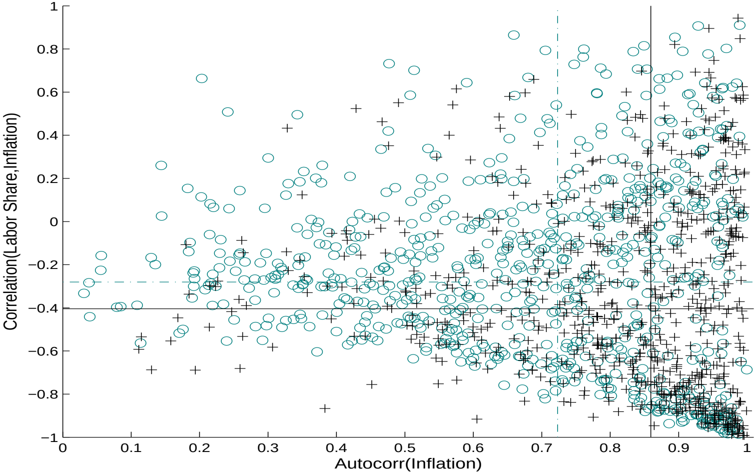
Questions:

1. Is there indexation?
2. Is $\widehat{\lambda}_{f,t}$ autocorrelated?

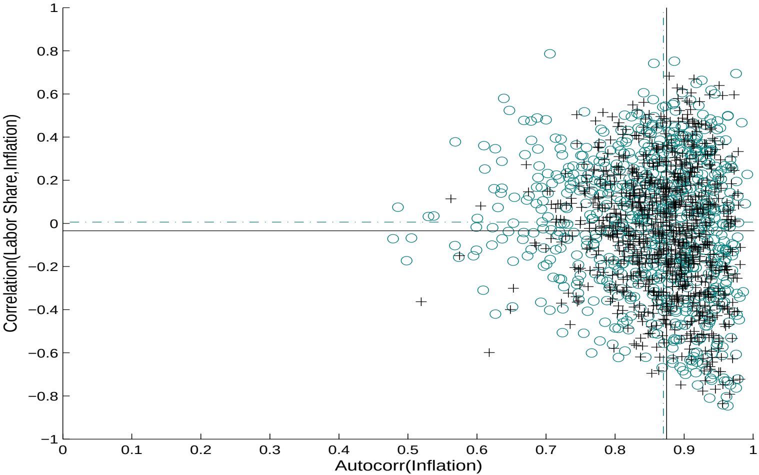
Indexation and the Phillips Curve

Standard Prior, No Indexation vs Benchmark



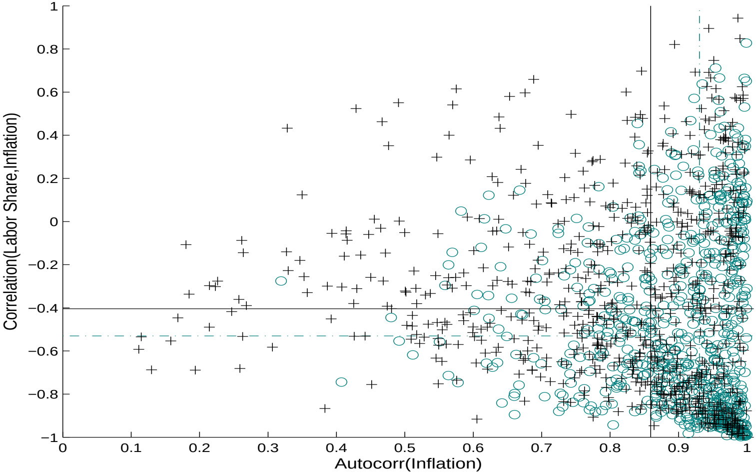
Indexation and the Phillips Curve

Dummy Obs. Prior, No Indexation vs Benchmark



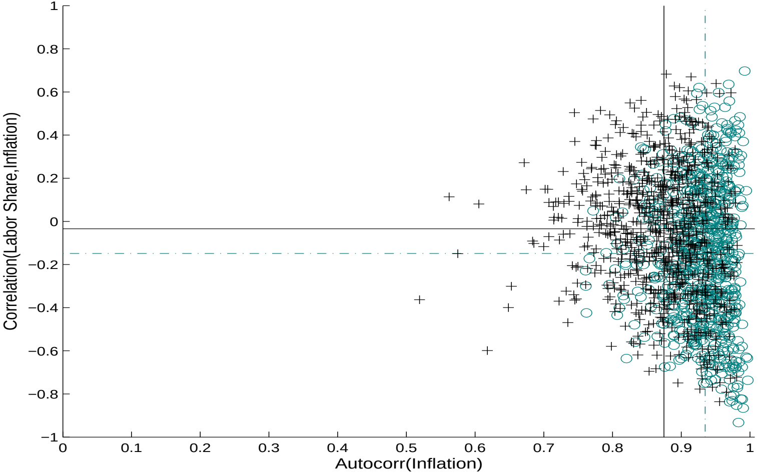
Indexation and the Phillips Curve

Standard Prior, Full Indexation vs Benchmark



Indexation and the Phillips Curve

Dummy Obs. Prior, Full Indexation vs Benchmark



Priors for Auxiliary Parameters

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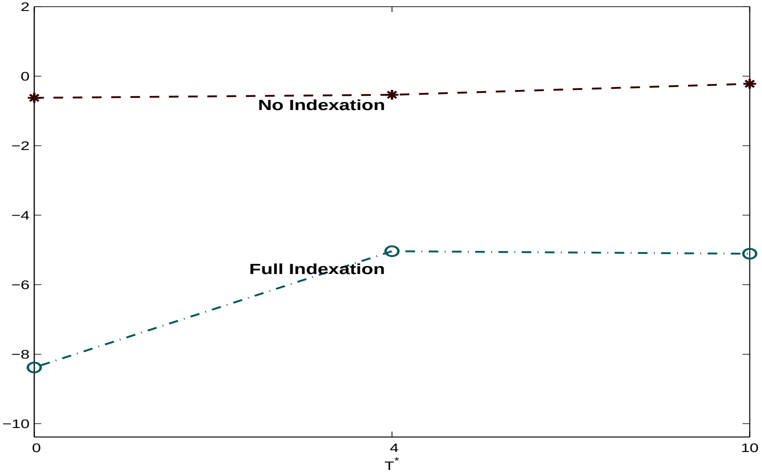
	Mean	StdDev	Mean	StdDev	Mean	StdDev	Mean	StdDev
ρ_z	0.400	0.250	0.489	0.129	0.490	0.125	0.459	0.127
ρ_ϕ	0.750	0.250	0.692	0.194	0.688	0.204	0.614	0.235
ρ_{λ_f}	0.750	0.250	0.843	0.120	0.872	0.089	0.838	0.130
ρ_g	0.750	0.250	0.597	0.278	0.625	0.287	0.573	0.280
σ_z	0.376	0.194	1.549	0.388	1.628	0.393	1.724	0.499
σ_ϕ	3.755	1.955	5.392	2.646	5.289	2.837	7.252	4.844
σ_{λ_f}	0.376	0.194	0.191	0.086	0.157	0.056	0.209	0.079
σ_g	0.626	0.323	0.577	0.204	0.570	0.241	0.543	0.195
σ_r	0.250	0.130	0.398	0.115	0.398	0.109	0.429	0.109

Marginal Likelihood Relative to Benchmark



Conclusion

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Findings:

- We need nominal rigidities – although for a different reason than the one empahsized in the literature.

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- Priors matter – using the same priors across different models may not be a good idea.
- Build a procedure to form priors for the auxiliary parameters such that these priors reflect our beliefs on the moments of the endogenous variables.

Findings:

- We need nominal rigidities – although for a different reason than the one empahsized in the literature.
- Do we need indexation? Models with no indexation and autocorrelated marginal shocks have the same fit of models likelihood as models with indexation and no autocorrelation in marginal shocks.