Estimating Macroeconomic Models: A Likelihood Approach

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- We apply particle filtering to evaluate the likelihood of the model.
- We estimate a neoclassical business cycle model with investment-specific technological change and stochastic volatility.

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- Alternatives? Problems?

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- We want to track conditional density $p\left(S_t | y^{t-1}; \gamma\right)$.

Factorization of the Likelihood

- Why?

$$
\begin{aligned}
p\left(y^{T};\gamma\right) &= \prod_{t=1}^{T} p\left(y_t|y^{t-1};\gamma\right) \\
&= \prod_{t=1}^{T} \int p\left(y_t|S_t;\gamma\right) p\left(S_t|y^{t-1};\gamma\right) dS_t
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- Knowledge of $\left\{p\left(S_t|y^{t-1};\gamma\right)\right\}_{t=1}^{T}$ allows the evaluation of the likelihood of the model.

A Law of Large Numbers

$$\prod_{t=1}^{T} \int p\left(y_t | S_t; \gamma\right) p\left(S_t | y^{t-1}; \gamma\right) dS_t$$

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Suppose we have $\left\{ \left\{ s_{t|t-1}^{i} \right\}_{i=1}^{N} \right\}_{t=1}^{T} \sim \left\{ p\left(S_t | y^{t-1}; \gamma\right) \right\}_{t=1}^{T}$.

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Then:

$$p\left(y^T; \gamma\right) \simeq \prod_{t=1}^{T} \frac{1}{N} \sum_{i=1}^{N} p\left(y_t | s_{t|t-1}^{i}; \gamma\right)$$

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Evaluating the likelihood function $\Leftrightarrow$ Drawing from density:

$$\left\{ p\left(S_t | y^{t-1} ; \gamma\right) \right\}_{t=1}^{T}$$

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2. Update: Bayes' theorem

$$p\left(S_t|y^{t};\gamma\right) = \frac{p\left(y_t|S_t;\gamma\right) p\left(S_t|y^{t-1};\gamma\right)}{p\left(y_t|y^{t-1};\gamma\right)}$$

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1. Forecast: Using a draw from $p\left(S_{t-1}|y^{t-1};\gamma\right)$, $p\left(W_t;\gamma\right)$, and $S_t = f\left(S_{t-1}, W_t;\gamma\right)$ we get a draw $p\left(S_t|y^{t-1};\gamma\right)$.

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2. Update: How do we draw from $p\left(S_t|y^t;\gamma\right)$ using draws from $p\left(S_t|y^{t-1};\gamma\right)$?.

Some Notation

- $\left\{ s^i_{t-1|t-1} \right\}_{i=1}^{N}$ N i.i.d. draws from $p\left(S_{t-1}|y^{t-1};\gamma\right)$.

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- Weights of draws $\left\{s_{t|t-1}^{i}\right\}_{i=1}^{N}$:

$$q_t^i = \frac{p\left(y_t|s_{t|t-1}^{i};\gamma\right)}{\sum_{i=1}^{N} p\left(y_t|s_{t|t-1}^{i};\gamma\right)}$$

A Proposition

- Let $\left\{ s_{t|t-1}^{i} \right\}_{i=1}^{N}$ be a draw from $p\left(S_t | y^{t-1}; \gamma\right)$.

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- Let $\left\{ s^i_{t|t-1} \right\}_{i=1}^N$ be a draw from $p\left(S_t|y^{t-1};\gamma\right)$.

- Let $\{\widetilde{s}^i_t\}_{i=1}^N$ be a draw with replacement from $\left\{ s^i_{t|t-1} \right\}_{i=1}^N$ and probabilities q^i_t.

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- Then, $\left\{\widetilde{s}_{t}^{i}\right\}_{i=1}^{N}$ is a draw from $p\left(S_t|y^{t};\gamma\right)$:

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- Proof: Importance sampling and Bayes' theorem.

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1. Forecast: We can use a draw $\left\{s_{t|t}^{i}\right\}_{i=1}^{N}$ from $p\left(S_t|y^t;\gamma\right)$, a draw from $p\left(W_{t+1};\gamma\right)$, and $S_{t+1}=f\left(S_t,W_{t+1};\gamma\right)$ to get a draw $\left\{s_{t+1|t}^{i}\right\}_{i=1}^{N}$.

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2. Update: Using the proposition, we can use a draw $\left\{s^i_{t|t-1}\right\}_{i=1}^{N}$ from $p\left(S_t|y^{t-1};\gamma\right)$ to get a draw $\left\{s^i_{t|t}\right\}_{i=1}^{N}$ from $p\left(S_t|y^t;\gamma\right)$.

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Step 0, Initialization: Sample N values $\left\{s_{1|0}^{i}\right\}_{i=1}^{N}$ from $p\left(S_1;\gamma\right)$. Go to step 2.

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Step 3, Update: Draw $\left\{s_{t|t}^{i}\right\}_{i=1}^{N}$ with replacement from $\left\{s_{t|t-1}^{i}\right\}_{i=1}^{N}$ with probabilities $\left\{q_t^i\right\}_{i=1}^{N}$. If $t < T$ set $t \rightsquigarrow t+1$ and go to step 1. Otherwise stop.

Particle Filtering II

Use $\left\{\left\{s^i_{t|t-1}\right\}_{i=1}^{N}\right\}_{t=1}^{T}$ to compute:

$$p\left(y^T;\gamma\right) \simeq \prod_{t=1}^{T}\frac{1}{N}\sum_{i=1}^{N} p\left(y_t|s^i_{t|t-1};\gamma\right)$$

We can filter, forecast, and smooth

An Application: a Business Cycle Model

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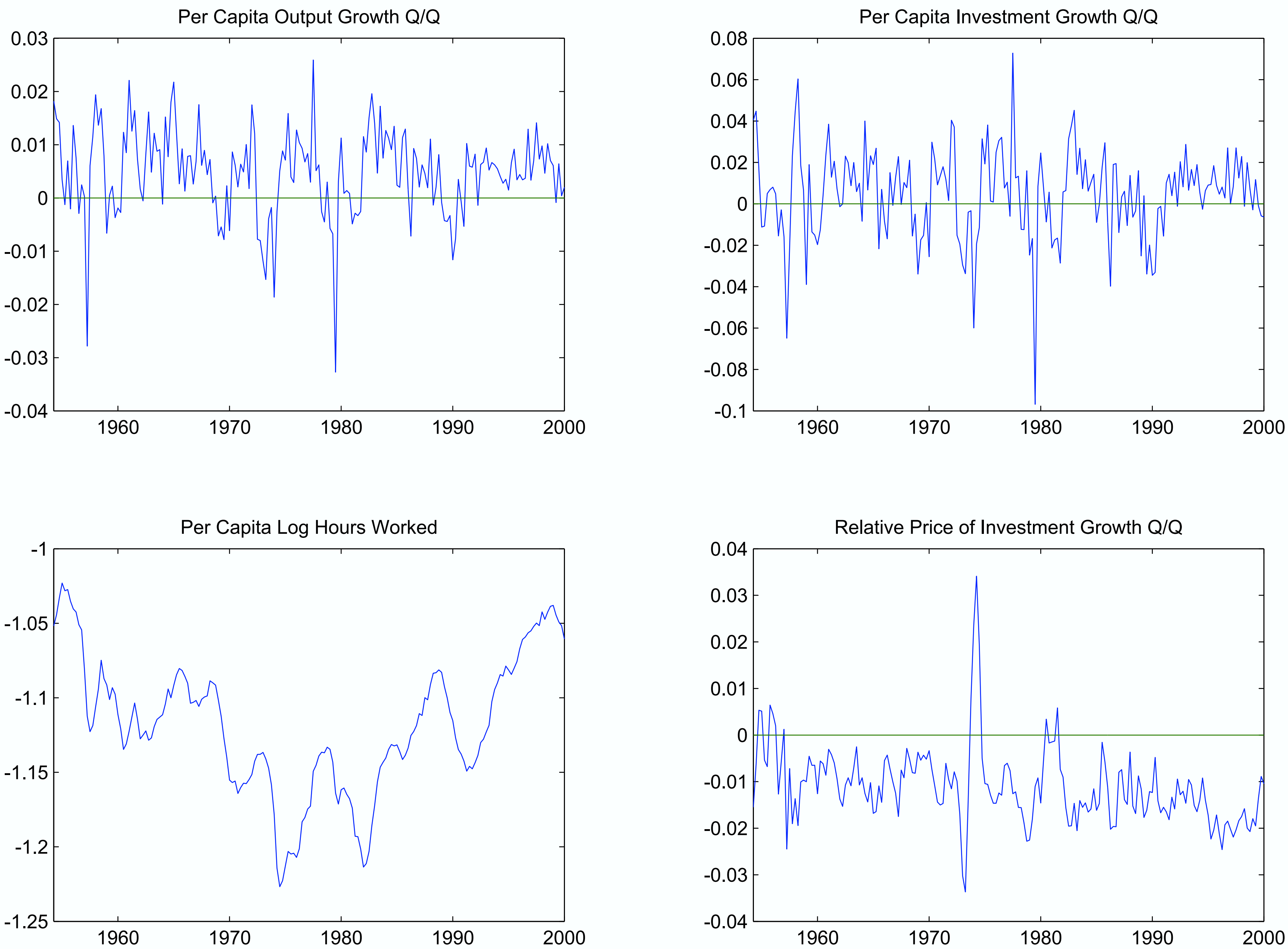
- I want to show the power of the particle filter.
- And learn something about the evolution of the U.S. aggregate fluctuations over the last decades.
- A business cycle model with:

1. Investment-specific technological change. Greenwood, Herkowitz, and Krusell (1997 and 2000)
2. Stochastic volatility.



Per Capita Output Growth Q/Q
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
1960
1970
1980
1990
2000
Per Capita Investment Growth Q/Q
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
1960
1970
1980
1990
2000
Per Capita Log Hours Worked
-1
-1.05
-1.1
-1.15
-1.2
-1.25
1960
1970
1980
1990
2000
Relative Price of Investment Growth Q/Q
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
1960
1970
1980
1990
2000

Environment

- Representative household with utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(e^{d_t} \log C_t + \psi \log (1 - L_t) \right)$$

- Final Good: $C_t + X_t = A_t K_t^{\alpha} L_t^{1-\alpha}$
- Law of motion of capital: $K_{t+1} = (1 - \delta) K_t + V_t X_t$
- Shocks:

$$\begin{aligned} d_t &= \rho d_{t-1} + \sigma_{dt} \varepsilon_{dt},\ \varepsilon_{dt} \sim \mathcal{N}(0,1) \\ \log A_t &= \gamma + \log A_{t-1} + \sigma_{at} \varepsilon_{at},\ \gamma \geq 0 \text{ and } \varepsilon_{at} \sim \mathcal{N}(0,1) \\ \log V_t &= \upsilon + \log V_{t-1} + \sigma_{\upsilon t} \varepsilon_{\upsilon t},\ \upsilon \geq 0 \text{ and } \varepsilon_{\upsilon t} \sim \mathcal{N}(0,1) \end{aligned}$$

Stochastic Volatility

We follow a standard specification:

$$\log \sigma_{dt} = (1 - \lambda_d) \log \overline{\sigma}_d + \lambda_d \log \sigma_{dt-1} + \tau_d \eta_{dt} \text{ and } \eta_{dt} \sim \mathcal{N}(0,1)$$

$$\log \sigma_{at} = (1 - \lambda_a) \log \overline{\sigma}_a + \lambda_a \log \sigma_{at-1} + \tau_a \eta_{at} \text{ and } \eta_{at} \sim \mathcal{N}(0,1)$$

$$\log \sigma_{\upsilon t} = (1 - \lambda_\upsilon) \log \overline{\sigma}_\upsilon + \lambda_\upsilon \log \sigma_{\upsilon t-1} + \tau_\upsilon \eta_{\upsilon t} \text{ and } \eta_{\upsilon t} \sim \mathcal{N}(0,1)$$

Performing Likelihood-Based Inference

- We compute the model using a perturbation method.
- Time series:

  1. Relative price of capital, output, investment, and hours.
  2. Sample: 1955:Q1 to 2000:Q4.

- Vector of parameters γ is:

  $$(\rho, \beta, \psi, \alpha, \delta, \upsilon, \zeta, \tau_d, \tau_a, \tau_\upsilon, \overline{\sigma}_d, \overline{\sigma}_a, \overline{\sigma}_\upsilon, \lambda_a, \lambda_\upsilon, \lambda_d, \sigma_1^\epsilon, \sigma_2^\epsilon, \sigma_3^\epsilon)$$

- Use a Random-walk Metropolis-Hastings to explore the likelihood: Classical and Bayesian.

Table 5.1: Maximum Likelihood Estimates

Parameter	Point Estimate	Standard Error ($x10^{-3}$)
ρ	0.967	3.743
β	0.999	0.460
ψ	2.343	6.825
υ	8.960E-003	0.828
ζ	3.594E-005	2.254
τ_a	7.120E-002	1.589
τ_υ	7.772E-003	2.940
τ_d	5.653E-002	2.034
$\overline{\sigma}_a$	4.008E-004	0.692
$\overline{\sigma}_\upsilon$	8.523E-003	0.101
$\overline{\sigma}_d$	5.016E-003	2.344
λ_a	4.460E-002	6.788
λ_υ	0.998	8.248
λ_d	0.998	2.302
$\sigma_{1\epsilon}$	1.031E-005	0.424
$\sigma_{2\epsilon}$	1.024E-004	0.495
$\sigma_{3\epsilon}$	1.110E-005	0.082

Figure 6.1: Model versus Data

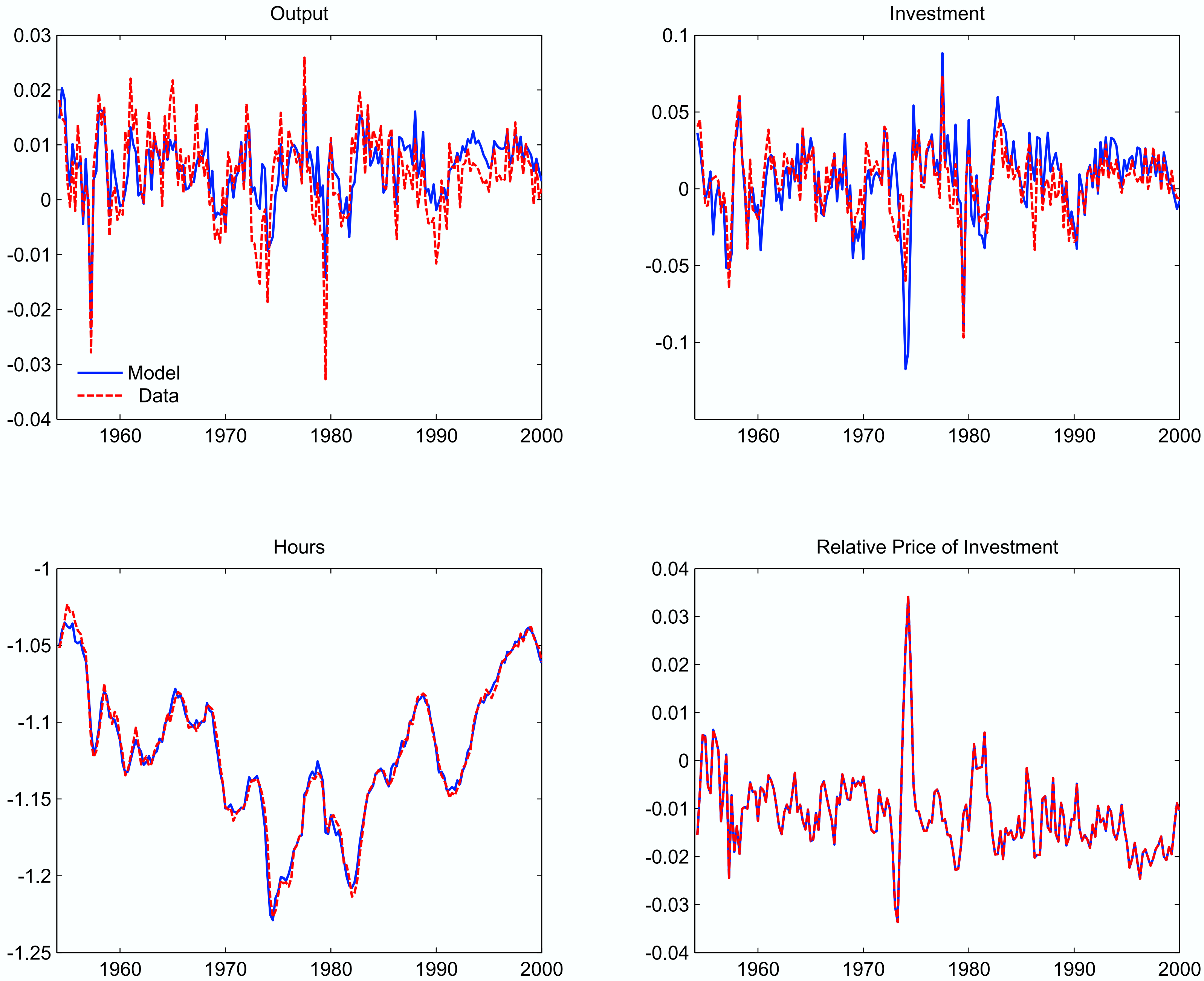


Figure 6.2: Smoothed Capital and Shocks

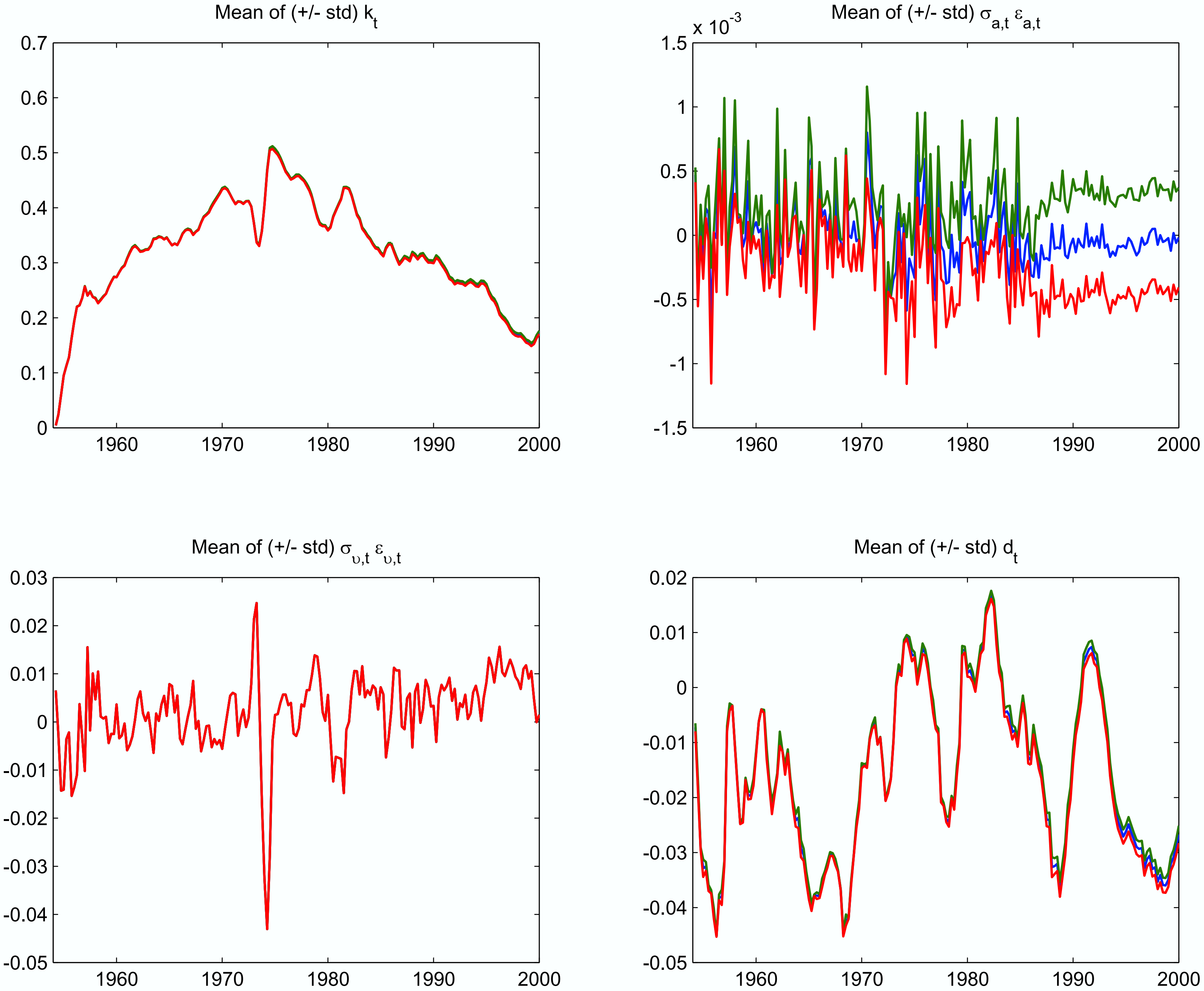


Figure 6.3: Smoothed Volatilities

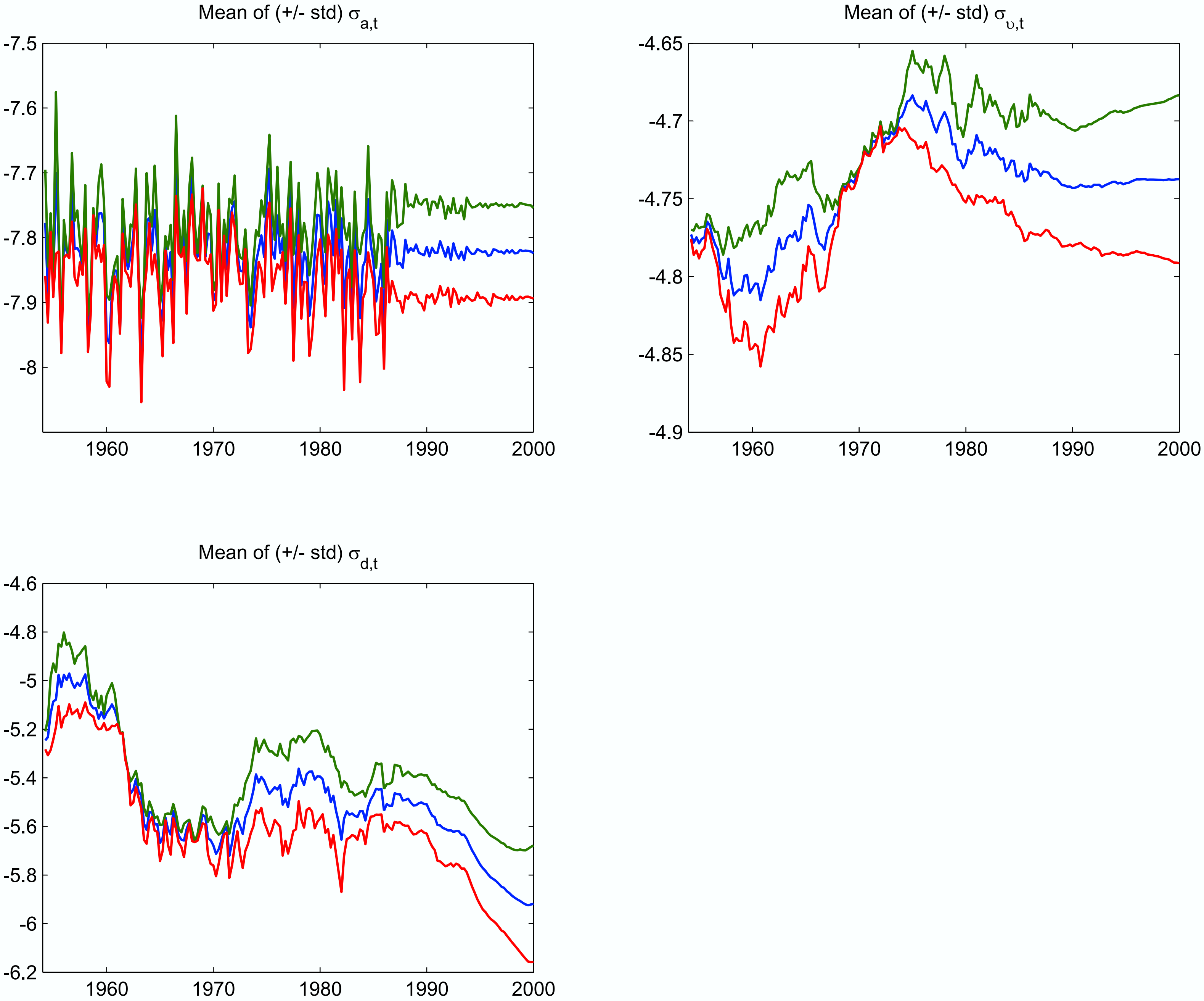


Figure 6.4: Instantaneous Standard Deviation

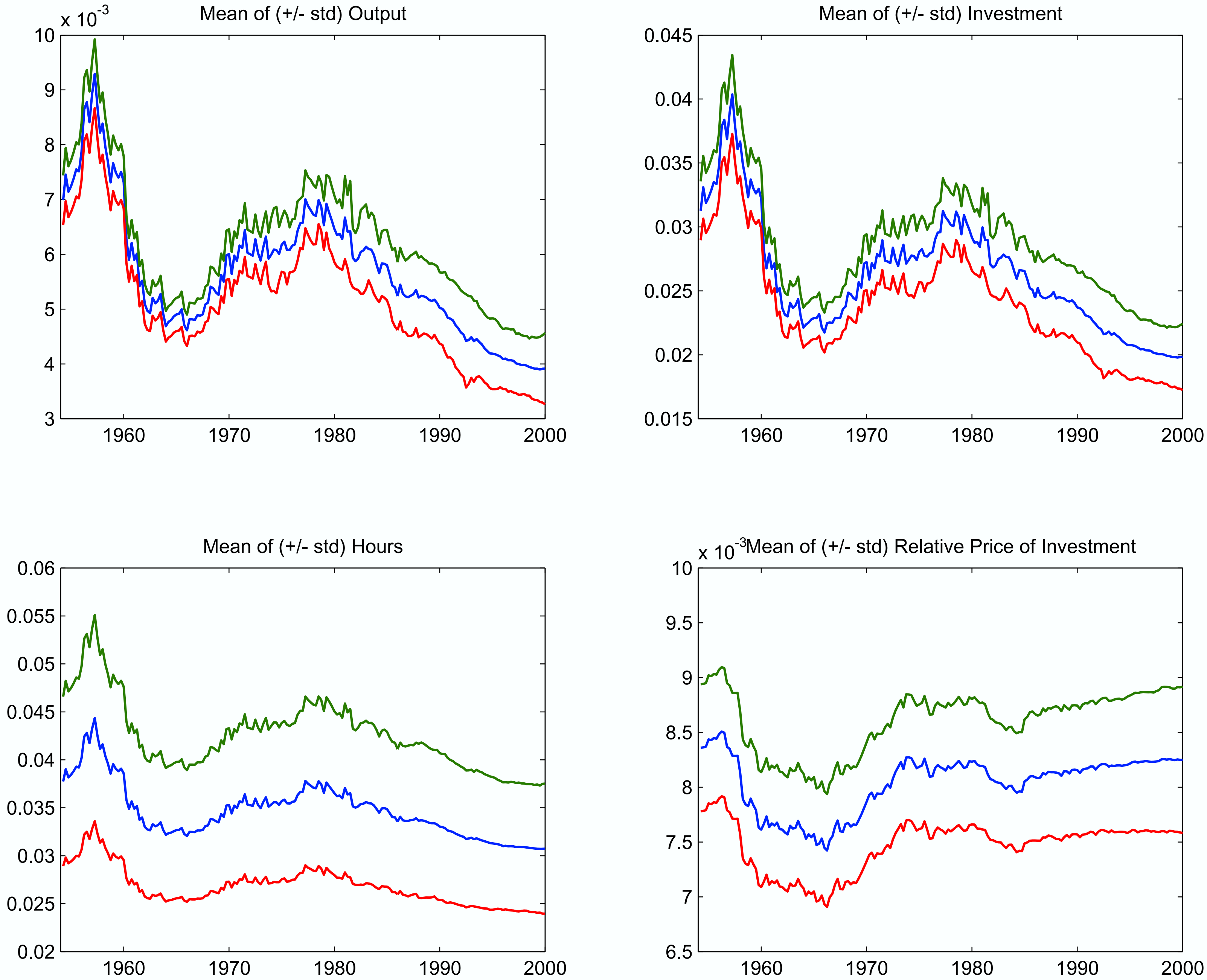


Figure 6.5: Counterfactual Exercise 1

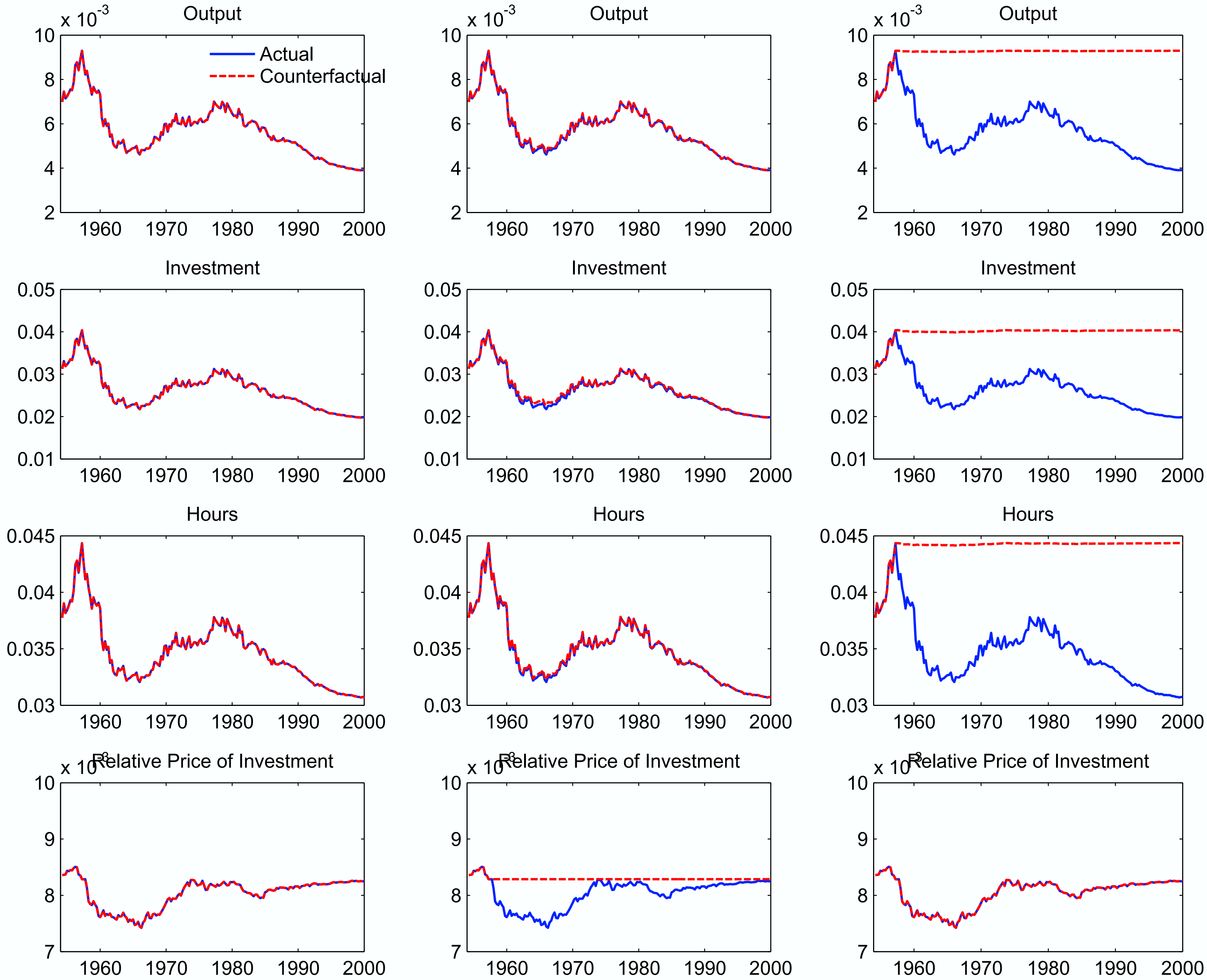


Figure 6.6: Counterfactual Exercise 2

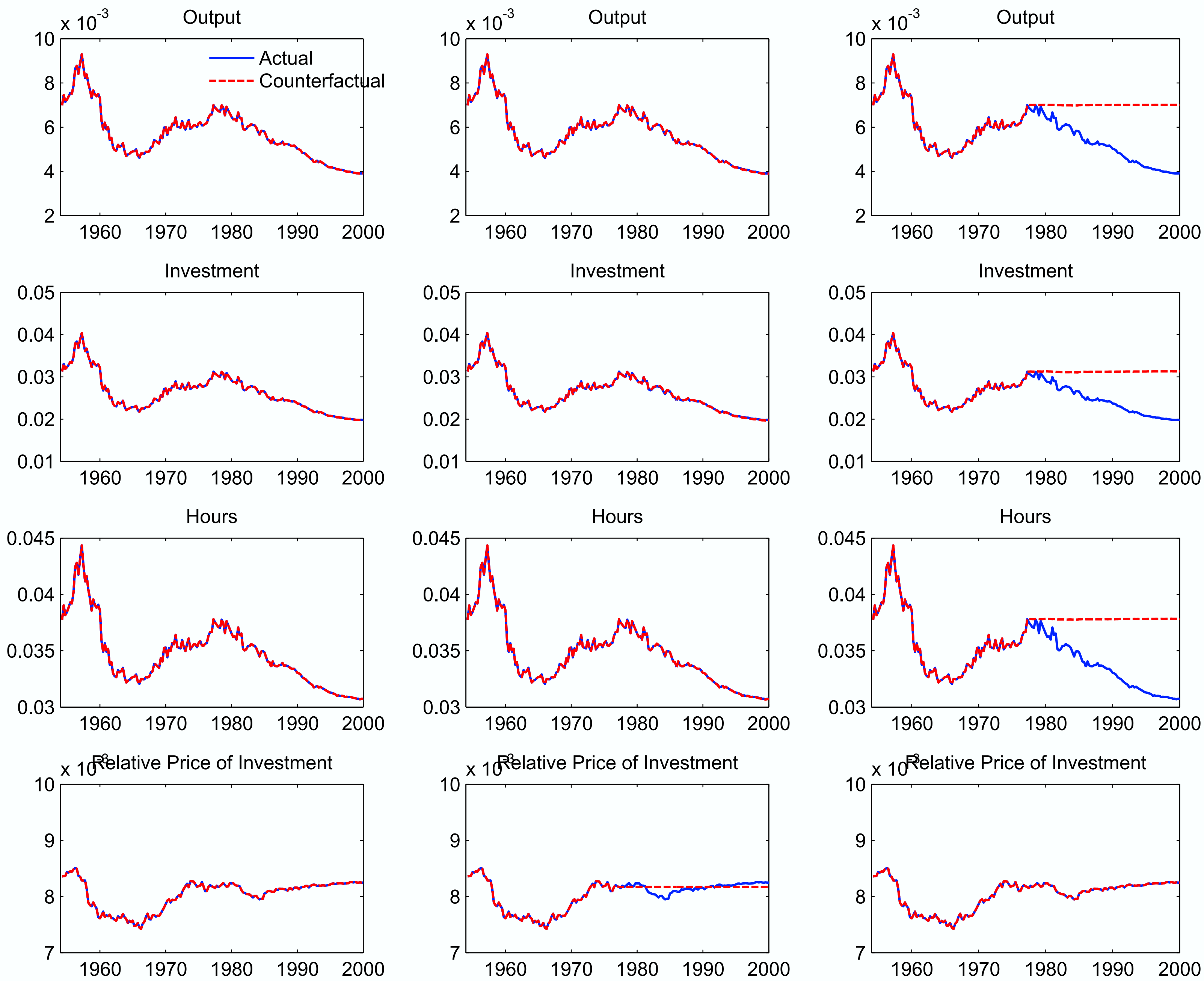
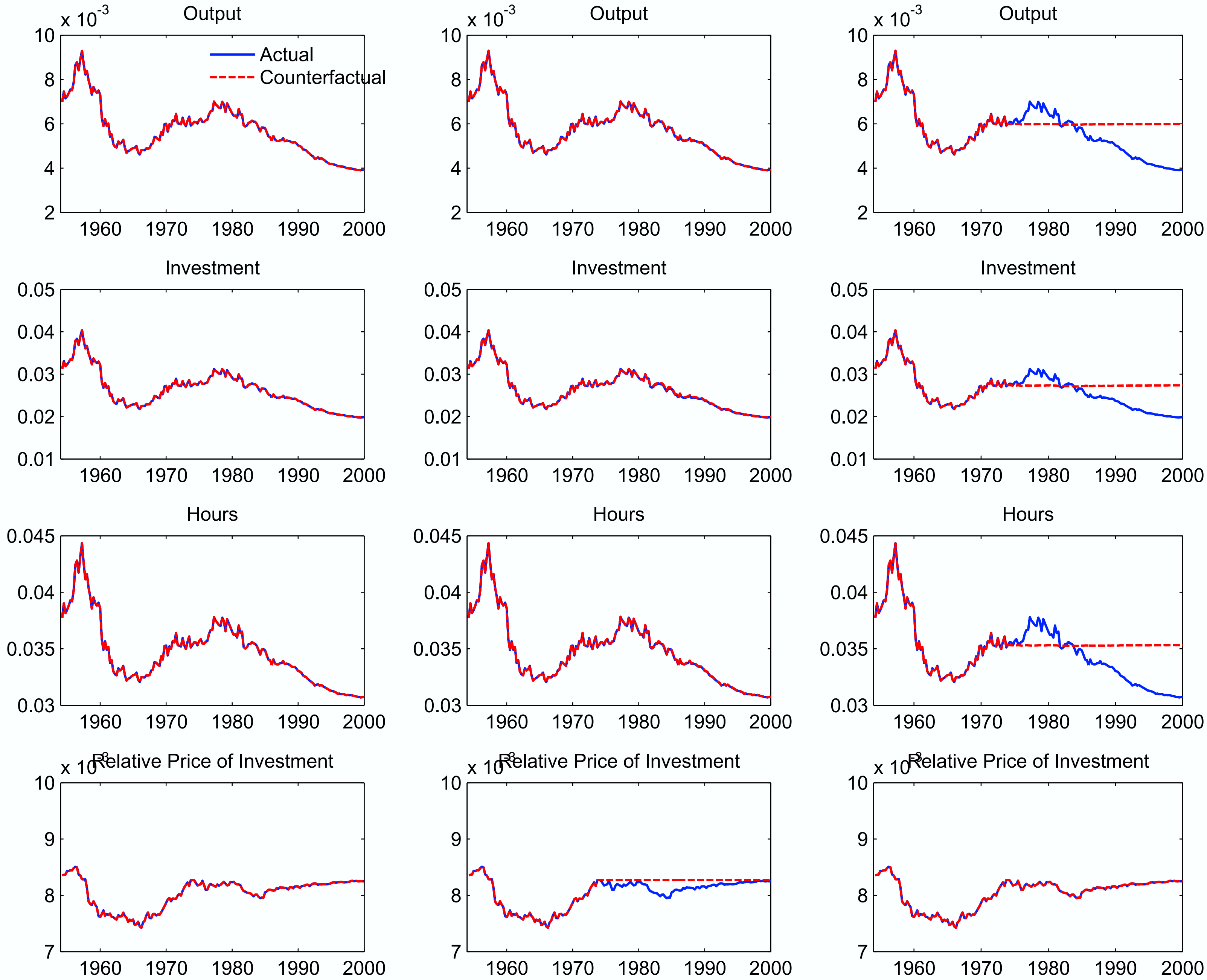


Figure 6.7: Counterfactual Exercise 3



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Linear	Version 1	Version 2
Quadratic	Version 3	Benchmark

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Solution	No Stochastic Volatility	Stochastic Volatility
Linear	Version 1	Version 2
Quadratic	Version 3	Benchmark

- We use Likelihood Ratio tests to compare models, Rivers and Vuong (2002).

Are Nonlinearities and Non-normalities Important?

- We estimate four version of the model:

Table 7.1: Versions of the Model

Solution	No Stochastic Volatility	Stochastic Volatility
Linear	Version 1	Version 2
Quadratic	Version 3	Benchmark

- We use Likelihood Ratio tests to compare models, Rivers and Vuong (2002).
- Loglike benchmark: 2350.6, loglike version 2: 2230.4

Figure 7.1: Comparison of Smoothed Capital and Shocks

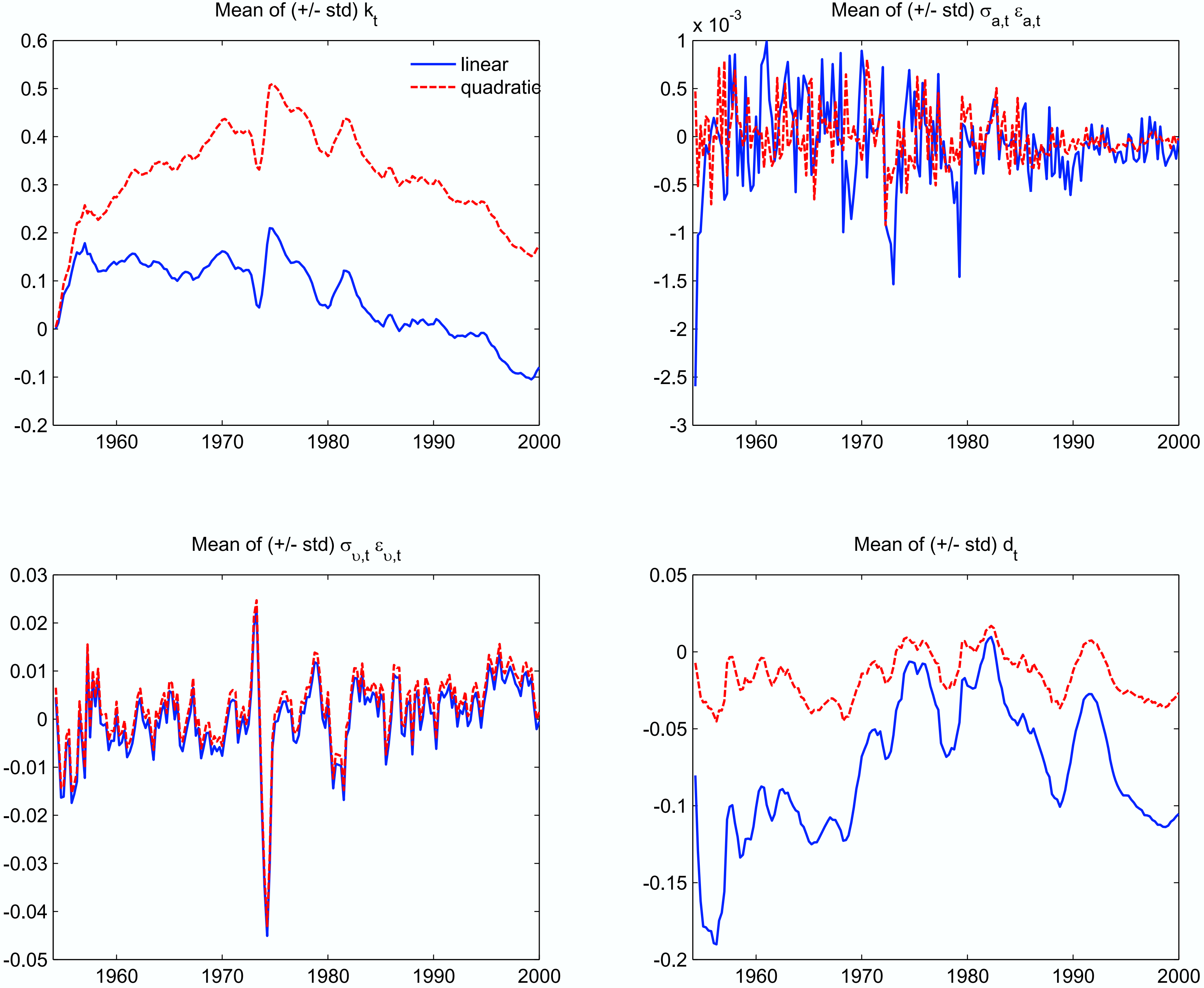
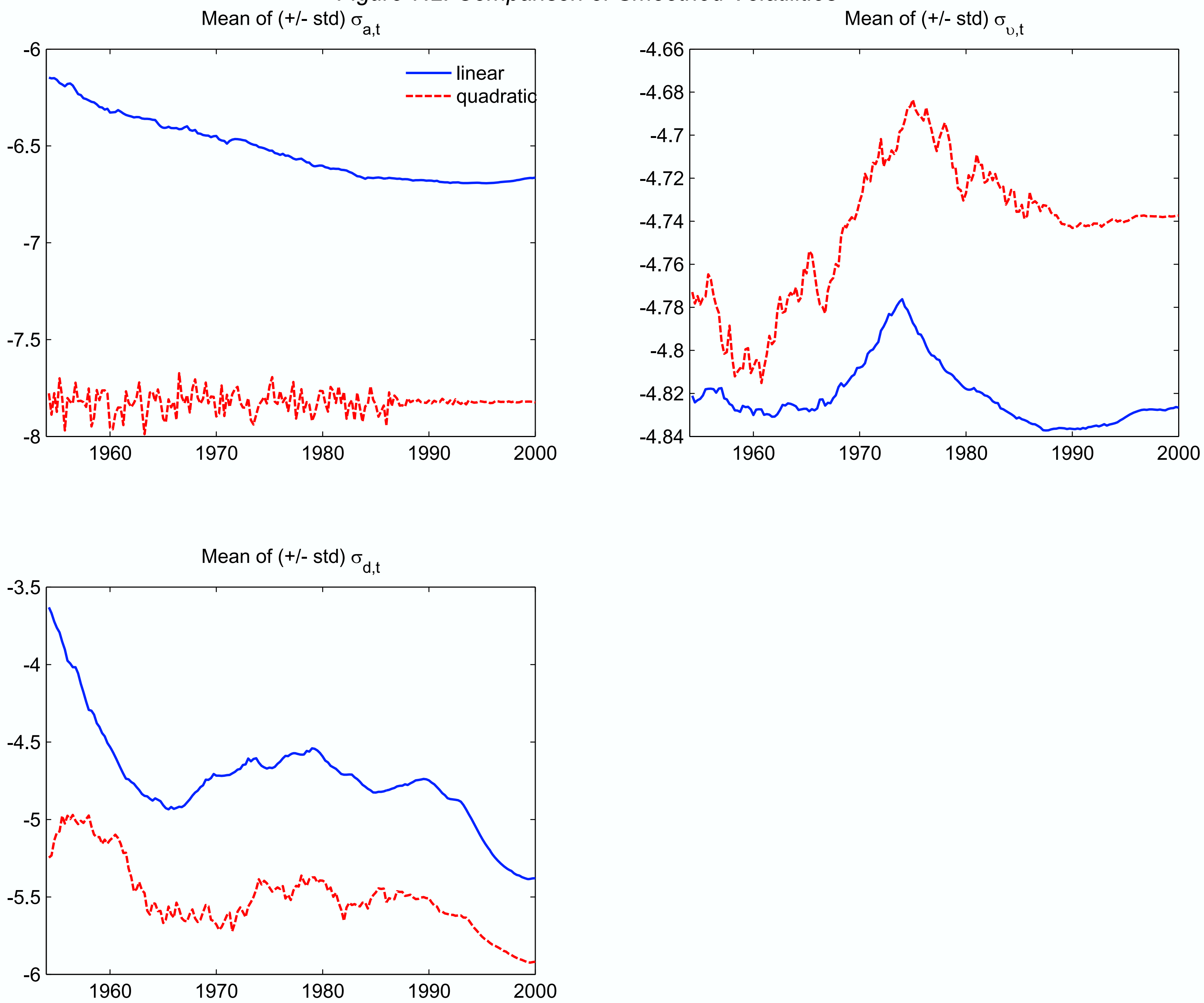


Figure 7.2: Comparison of Smoothed Volatilities



Conclusions

1. Particle filtering is a general purpose and efficient method to estimate DSGE models.

2. We learned about the importance of stochastic volatility to account for U.S. Business Cycle.

3. Much exciting work to do in the next few years!