THE PHILLIPS CURVE UNDER STATE-DEPENDENT PRICING

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Outline

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- IMPLICATIONS FOR INFLATION DYNAMICS
- EMPIRICAL ISSUES
- CONCLUDING REMARKS

CONTEXT

- Dominant approach in the literature: time-dependent pricing model $\grave{a}$ la Calvo which implies the New Keynesian Phillips Curve: (NKPC)

$$\pi_t = \beta E_t \pi_{t+1} + \lambda mc_t$$

- Empirical studies provide mixed results. NKPC Performance can be improved by adding a lagged inflation term $\longrightarrow$ hybrid NKPC:

$$\pi_t = \gamma_b \pi_{t-1} + \gamma_f E_t \pi_{t+1} + \lambda mc_t$$

- Problems
  - with hybrid NKPC: lagged inflation term is not micro-founded.
  - with time-dependent pricing models in general: timing of price adjustment is exogenously specified and therefore invariant to shifts in the macroeconomic environment.

CONTEXT (contd.)

- Alternative framework: State-dependent pricing
  - Timing of price adjustment is determined endogenously and therefore responding to shifts in the macroeconomic environment.
  - Early contributions by Sheshinsky and Weiss (1982, 1983) and Caplin and Leahy (1982). Tractable model by Dotsey, King and Wolman (1999).

DERIVATION OF THE SDPC

Model framework: Dotsey, King and Wolman (QJE 1999)

- Key element: explicit formulation of price adjustment costs;
- in each period each firm faces a fixed cost of adjusting its price, ξ_t (in labor hours) drawn from a continuous distribution function $G(\xi)$;
- this cost is random across firms and over time;
- realizations of adjustment cost are bounded from above $\longrightarrow$ guarantees a finite number of vintages in a positive inflation environment;
- otherwise standard DSGE framework (competitive factor markets, no capital accumulation).

DERIVATION OF THE SDPC (contd.)

With stochastic adjustment costs, the pricing problem has two aspects:

- determination of timing of price adjustment ($\neq$ Calvo model),
- determination of optimal nominal price

Firms maximize a dynamic programming problem:

$$V_t(P_{j,t}, \xi_t, s_t) = \max\left(v_{0,t} - \xi_t w_t, v_{j,t}\right)$$

$$v_{0,t} = \max_{P_{0,t}}\left[z_{0,t} + E_t\beta Q_{t,t+1}V_{t+1}(P_{1,t+1}, \xi_{t+1}, s_{t+1})\right]$$

$$v_{j,t} = z_{j,t} + E_t\beta Q_{t,t+1}V_{t+1}(P_{j+1,t+1}, \xi_{t+1}, s_{t+1})$$

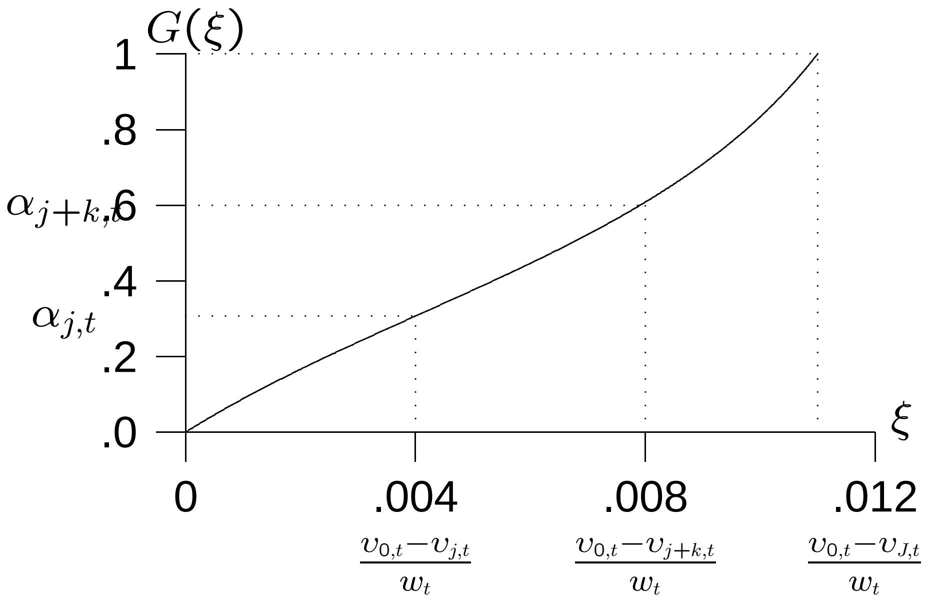
$\xi_t w_t$: realization of adjustment costs in t.

DERIVATION OF THE SDPC (contd.)

Adjustment probabilities are endogenous in the DKW model

Adjustment decision rule $\longrightarrow$ Adjustment probability of price vintage j

$$v_{0,t} - v_{j,t} \geq w_t \xi_t \qquad\qquad \alpha_{j,t} = G\left(\frac{v_{0,t} - v_{j,t}}{w_t}\right)$$



DERIVATION OF THE SDPC (contd.)

Optimal nominal price, $P_{0,t}$, satisfies Euler condition that involves balancing pricing effects on current and expected future profits:

$$0 = E_t \sum_{j=0}^{J-1} \beta^j Q_{t,t+j} \frac{\omega_{j,t+j}}{\omega_{0,t}} \frac{\partial z_{j,t+j}}{\partial P_{0,t}},$$

where $\omega_{0,t} = \sum_{j=1}^{J} \alpha_{j,t} \omega_{j-1,t-1}$ and $\omega_{j,t} = (1 - \alpha_{jt}) \omega_{j-1,t-1}$

Optimal nominal price set by adjusting firms, $P_{0,t}$, and aggregate price level, P_t:

$$P_{0,t} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{j=0}^{J-1} \beta^j Q_{t,t+j} \frac{\omega_{j,t+j}}{\omega_{0,t}} MC_{t+j} P_{t+j}^{\theta} C_{t+j}}{E_t \sum_{j=0}^{J-1} \beta^j Q_{t,t+j} \frac{\omega_{j,t+j}}{\omega_{0,t}} P_{t+j}^{\theta-1} C_{t+j}}; \quad P_t \equiv \left[\sum_{j=0}^{J-1} \omega_{j,t} (P_{0,t-j})^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

DERIVATION OF THE SDPC (contd.)

Log-linearized optimal relative price, $x_{0,t}$:

$$x_{0,t} = E_t \sum_{j=1}^{J-1} \sum_{i=j}^{J-1} [\theta\rho_i + (1-\theta)\delta_i]\pi_{t+j} + E_t \sum_{j=0}^{J-1} \{\psi_j mc_{t+j} + (\rho_j - \delta_j)[\widehat{\omega}_{j,t+j} - \widehat{\omega}_{0,t}]\} \quad (1)$$

with

$$\rho_j = \frac{\beta^j \omega_j \Pi^{j\theta}}{\sum_{i=0}^{J-1} \beta^i \omega_i \Pi^{i\theta}}, \qquad \delta_j = \frac{\beta^j \omega_j \Pi^{j(\theta-1)}}{\sum_{i=0}^{J-1} \beta^i \omega_i \Pi^{i(\theta-1)}}, \qquad \psi_j = \rho_j + \kappa(\rho_j - \delta_j)$$

Log-linearized aggregate price level in terms of optimal relative price, $x_{0,t}$:

$$x_{0,t} = \mu_0 \pi_t + \sum_{j=1}^{J-2} \mu_j \pi_{t-j} - \sum_{j=1}^{J-1} \omega_j \nu_j x_{0,t-j} - \frac{1}{1-\theta} \sum_{j=0}^{J-1} \nu_j \widehat{\omega}_{j,t} \quad (2)$$

$$\mu_j = \frac{1}{\omega_0} \sum_{i=j+1}^{J-1} \omega_i \Pi^{i(\theta-1)}, \qquad \nu_j = \frac{1}{\omega_0} \Pi^{j(\theta-1)}$$

DERIVATION OF THE SDPC (contd.)

The state-dependent Phillips curve (SDPC):

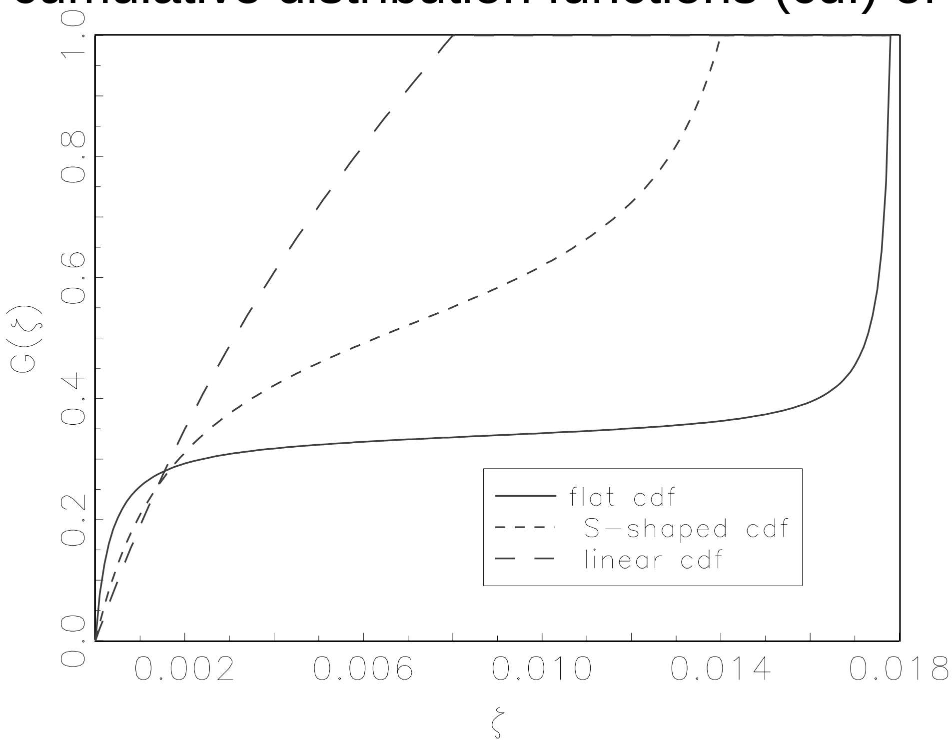
$$\pi_t = E_t \sum_{j=1}^{J-1} \delta_j' \pi_{t+j} + E_t \sum_{j=0}^{J-1} \psi_j' mc_{t+j} + E_t \sum_{j=0}^{J-1} \gamma_j [\widehat{\omega}_{j,t+j} - \widehat{\omega}_{0,t}] + \sum_{j=0}^{\infty} \eta_j \widehat{\Omega}_{t-j} + \sum_{j=1}^{\infty} \mu_j' \pi_{t-j}$$

- General representation of inflation dynamics
- SDPC collapses to the NKPC if $\alpha_{j,t} = \alpha$ and $\pi_{ss} = 0\%$
- Reduced-form parameters respond to shifts in trend inflation, degree of competition etc.

CHARACTERIZATION OF THE SDPC (contd.)

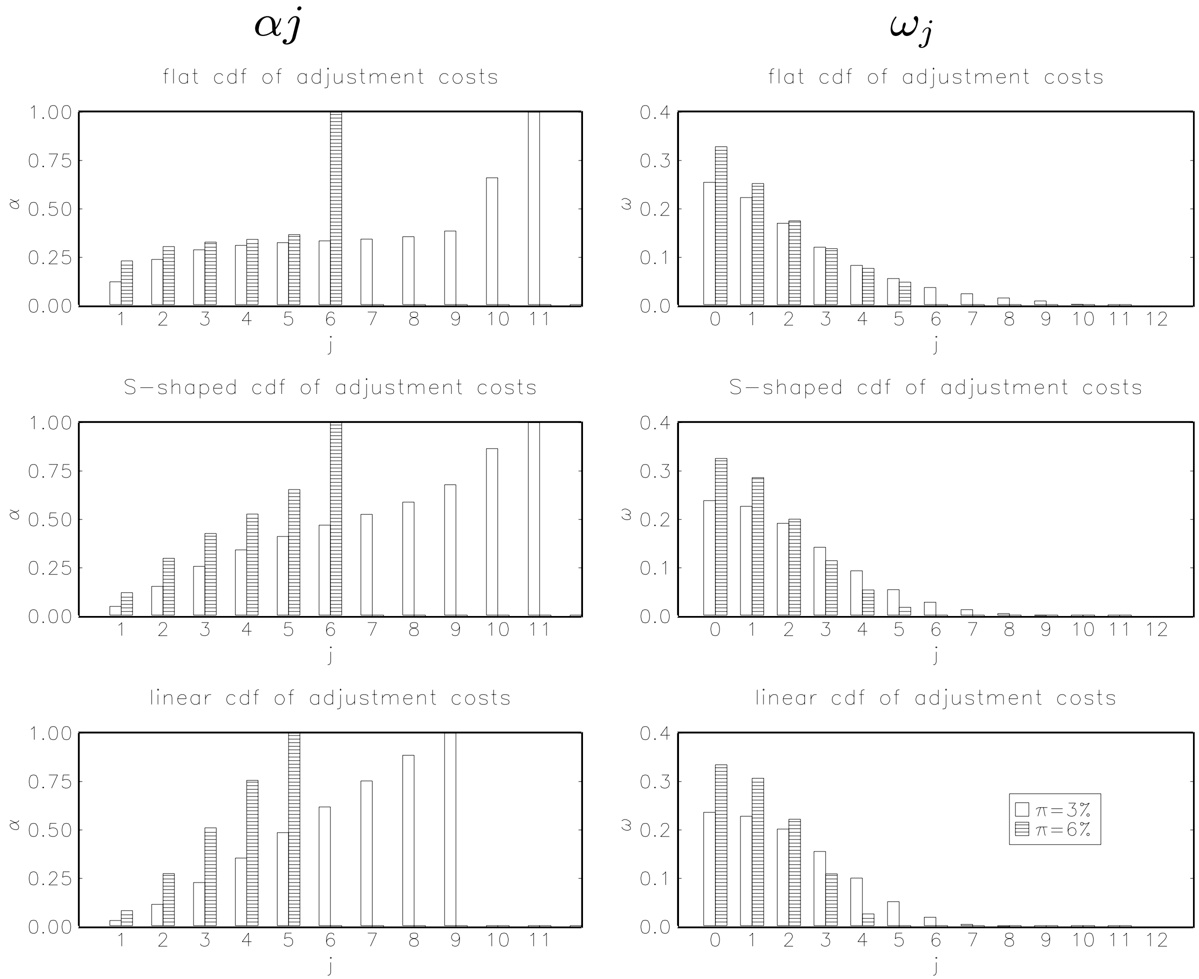
Sensitivity of SDPC parameters to changes in the exogenous assumptions:

- 2 levels of trend inflation: $3\%, 6\%$
- 3 types of cumulative distribution functions (cdf) of adjustment costs:



CHARACTERIZATION OF THE SDPC (contd.)

Price-setting behavior along steady state

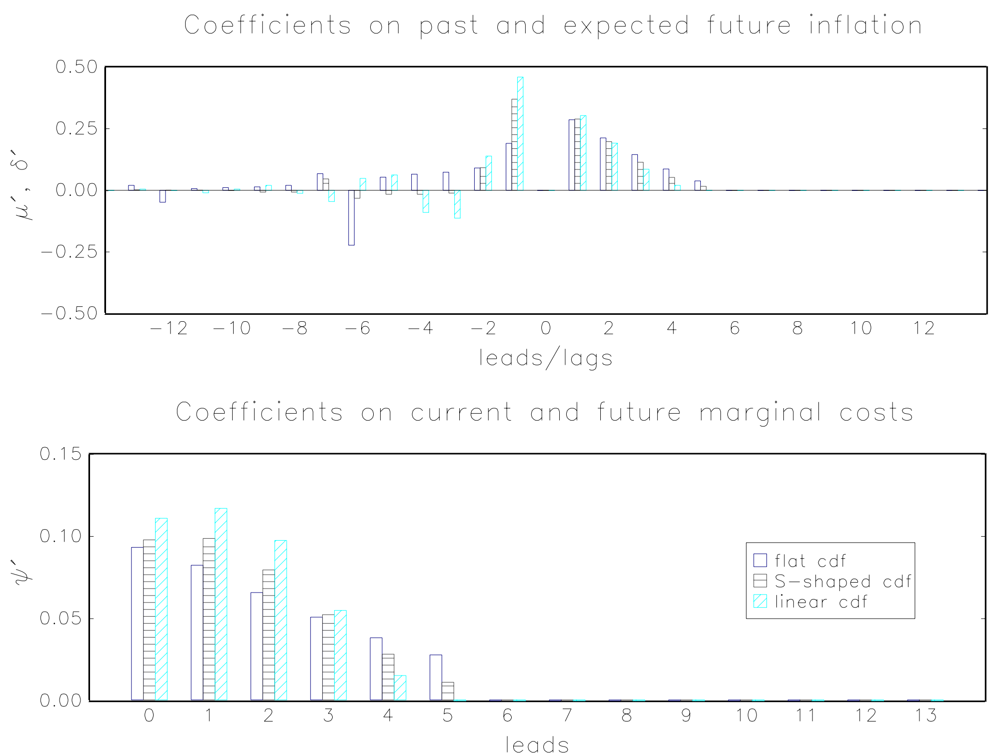


CHARACTERIZATION OF THE SDPC (contd.)

Reduced form SDPC coefficients

$\pi_{ss} = 3\%$

$\pi_{ss} = 6\%$



CHARACTERIZATION OF THE SDPC (contd.)

SDPC and intrinsic persistence

	$\pi_{ss} = 3\%$			$\pi_{ss} = 6\%$		
cdf	μ'_1	$\sum_{j=1}^{3J} \mu'_j$	D	μ'_1	$\sum_{j=1}^{3J} \mu'_j$	D
flat	0.198	0.297	3.9	0.190	0.341	3.0
S-shaped	0.280	0.409	4.2	0.370	0.421	3.1
linear	0.303	0.453	4.2	0.459	0.465	3.0

Notes: D = average duration of price stickiness in quarters.

IMPLICATIONS FOR INFLATION DYNAMICS

To close the model, we adopt a standard monetary policy rule of the form

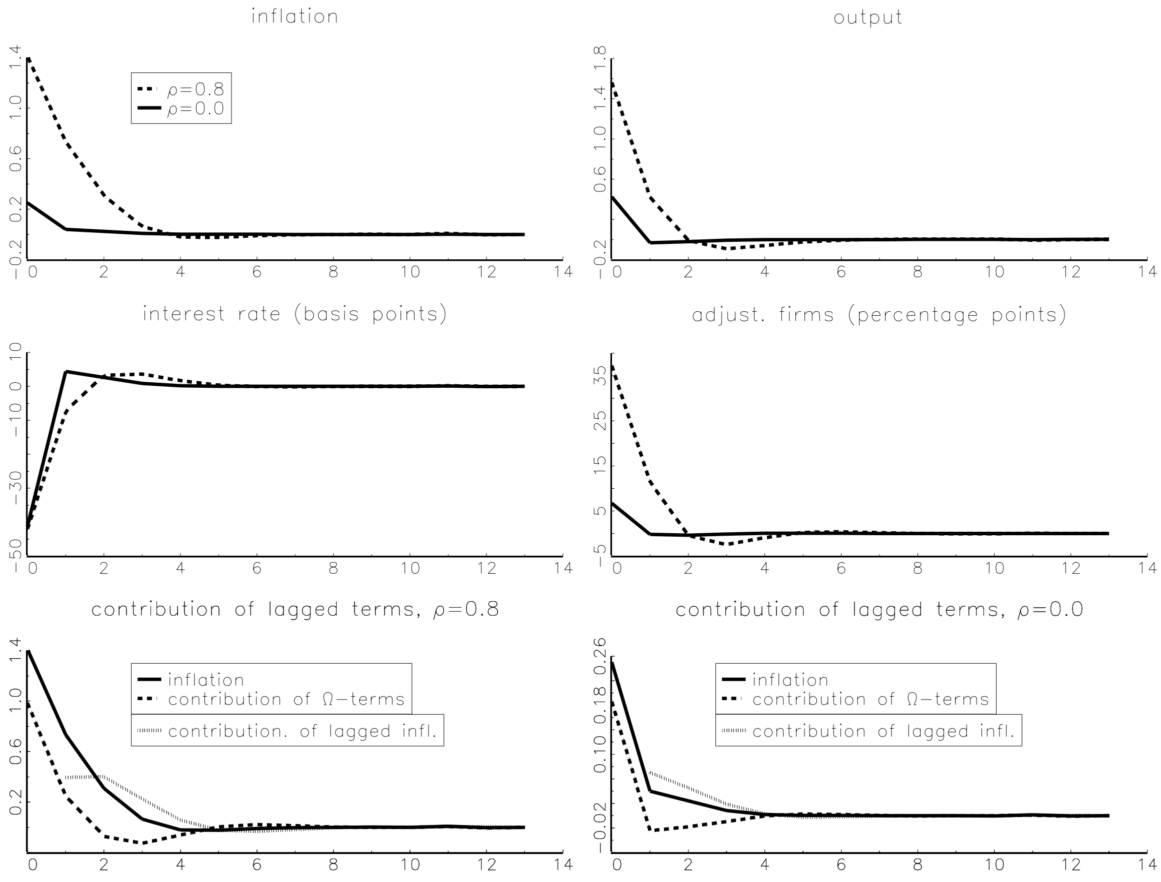
$$i_t = \rho i_{t-1} + (1-\rho)(1.5\pi_t + 0.5 y_t) + \epsilon_t$$

Model calibrations

Quarterly discount factor	β = 0.984
Risk aversion	σ = 1
Labor supply elasticity	∞
Labor share	α_L = 0.667
Demand elasticity	θ = 10
Adjustment costs cdf	S-shaped
Steady-state inflation	3%
Monetary policy inertia	ρ = 0.8, 0

IMPLICATIONS FOR INFLATION DYNAMICS (contd.)

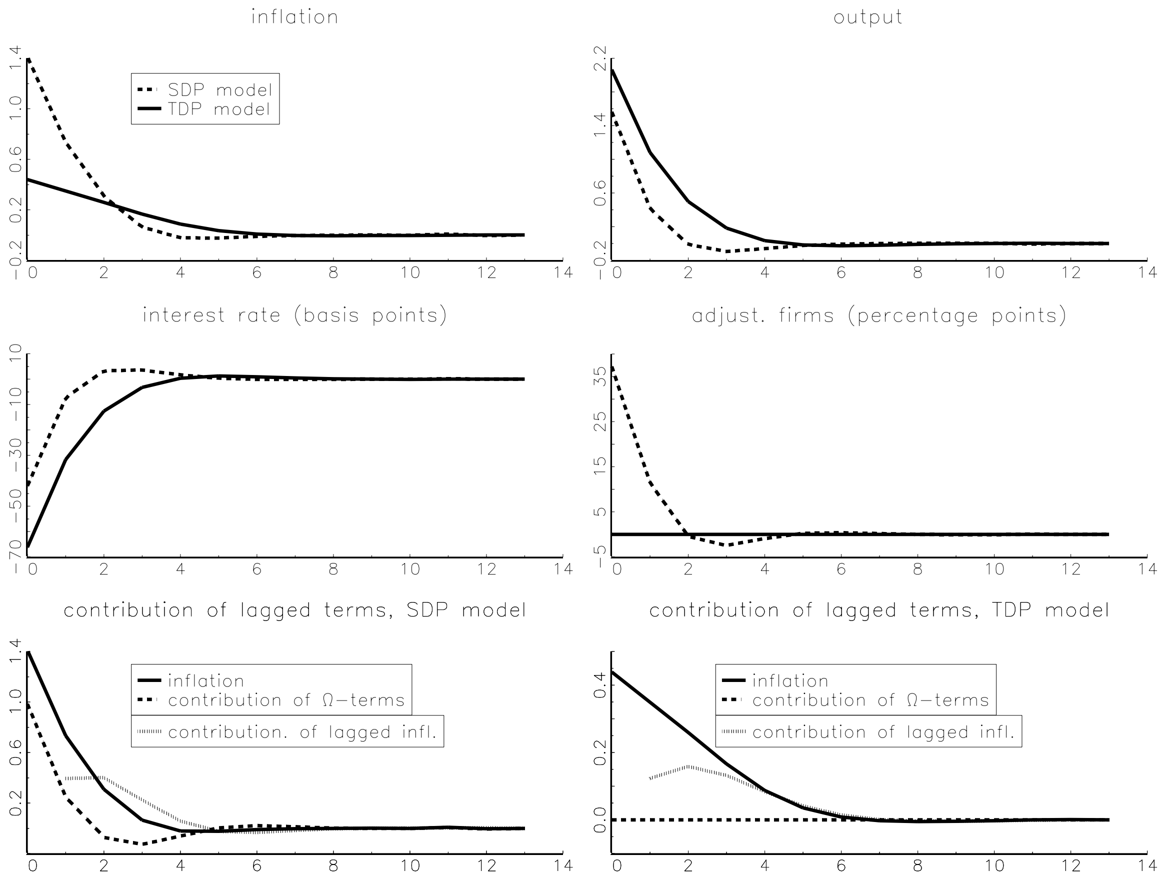
Effect of policy regime ($\rho = 0.8$ vs. $\rho = 0$) on inflation dynamics



Responses to an expansionary interest rate shock, $\pi_{SS} = 3\%$ and S-shaped cdf

IMPLICATIONS FOR INFLATION DYNAMICS (contd.)

Effect of state-dependent variations in price setting on inflation dynamics



Responses to an expansionary interest rate shock, $\pi_{SS} = 3\%$, $\rho = 0.8$, and S-shaped cdf

EMPIRICAL ISSUES

Are we misled by the hybrid NKPC?

EXPERIMENT:

- Simulations of data sets (1000 data sets, 150 observations) in the state-dependent model of price stickiness (3 types of shocks),
- estimation of the hybrid NKPC (GMM),
- incorporation of estimated hybrid NKPC in a small New Keynesian model to generate dynamic responses to a monetary policy shock,
- comparison of dynamic responses in the “true” model with those from the small Keynesian model (with estimated hybrid NKPC) allows assessment of the hybrid NKPC.

Estimation results for the hybrid NKPC

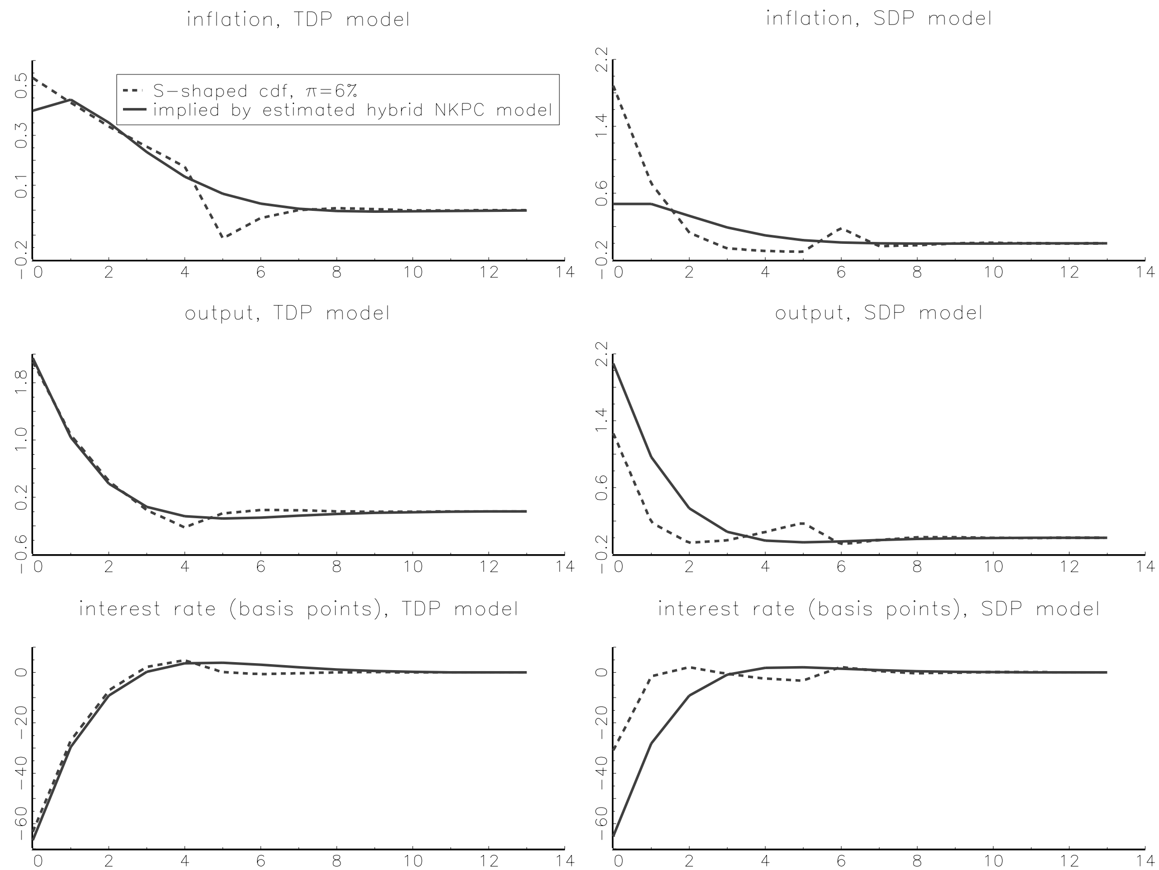
$$\pi_t = \gamma_b \pi_{t-1} + \gamma_f E_t \pi_{t+1} + \lambda mc_t$$

π_{ss}	Model	$\widehat{\lambda}$	$\widehat{\gamma}_f$	$\widehat{\gamma}_b$	$\widehat{D}$
3%	tdp	0.027 (0.534)	0.574	0.426	3.8
		[0.000 , 0.063]	[0.464 , 0.710]		
3%	sdp	0.041 (0.304)	0.610	0.390	3.5
		[-0.009 , 0.103]	[0.514 , 0.728]		
6%	tdp	0.069 (0.794)	0.566	0.434	2.7
		[0.018 , 0.129]	[0.479 , 0.692]		
6%	sdp	0.088 (0.395)	0.614	0.386	2.7
		[-0.017 , 0.198]	[0.519 , 0.738]		

Notes: $\gamma_f + \gamma_b = 1$,
$\widehat{D}$ = estimated average duration of price stickiness,

EMPIRICAL ISSUES (contd.)

Performance of GMM estimate of HNKPC



Responses to an expansionary interest rate shock, $\pi_{SS} = 6\%$, $\rho = 0.8$

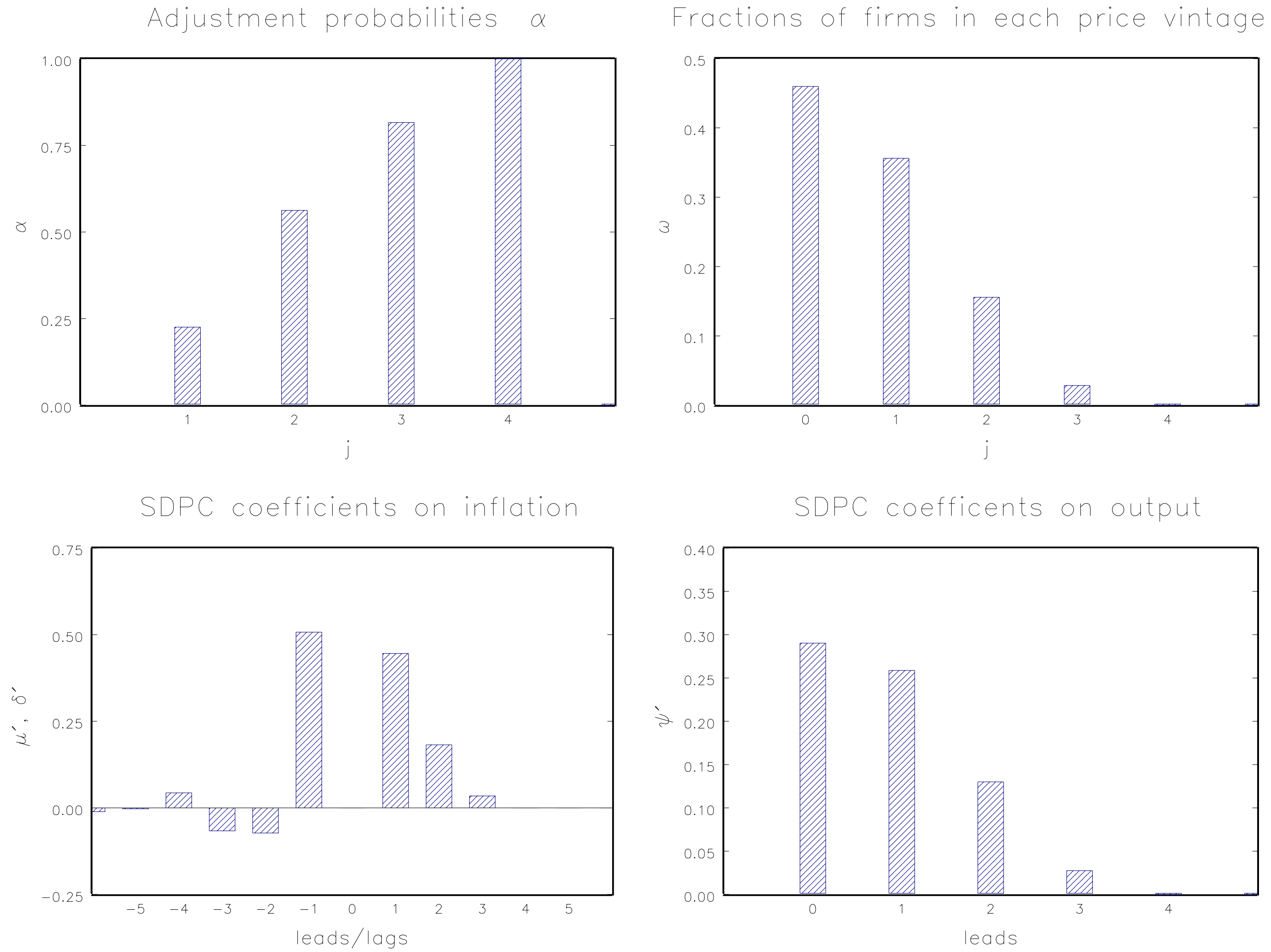
EMPIRICAL ISSUES (contd.)

Bayesian estimation of DSGE models with state-dependent pricing (not in the paper)

- Potential problems:
  - infinite number of lagged inflation terms and lagged Ω_{t-j}-terms $\longrightarrow$ truncation
  - finite (but potentially high) number of leads in inflation and output gap $\longrightarrow$ truncation
  - state-dependent variations in price setting,Ω_{t-j}, are not observable $\longrightarrow$ need to be estimated as unobserved component.
- Is truncation innocuous?
  - simulation of a data set in the DKW framework when the the number of price vintages is restricted to 4,
  - Bayesian estimation of a small DSGE model that incorporates the SDPC truncated at two different lag orders,
  - comparison of results.

EMPIRICAL ISSUES (contd.)

Comparison of M1L2L and M1L3L



EMPIRICAL ISSUES (contd.)

Small DSGE model with time-dependent pricing

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}) + g_t$$

$$\pi_t = E_t \sum_{j=1}^{3} \delta'_j \pi_{t+j} + E_t \sum_{j=0}^{3} \psi'_j mc_{t+j} + \sum_{j=1}^{\infty} \mu'_j \pi_{t-j} + z_t$$

M1L2L: $\pi_t = E_t \delta'_1 \pi_{t+j} + E_t \sum_{j=0}^{1} \psi'_j mc_{t+j} + \sum_{j=1}^{2} \mu'_j \pi_{t-j} + z_t$

M1L3L: $\pi_t = E_t \delta'_1 \pi_{t+j} + E_t \sum_{j=0}^{1} \psi'_j mc_{t+j} + \sum_{j=1}^{3} \mu'_j \pi_{t-j} + z_t$

$$i_t = \rho i_{t-1} + (1 - \rho)(\phi_\pi \pi_t + \phi_y y_t) + \epsilon_{R,t}$$

$$g_t = \rho_g g_{t-1} + \epsilon_{g,t}$$

$$z_t = \rho_z z_{t-1} + \epsilon_{z,t}$$

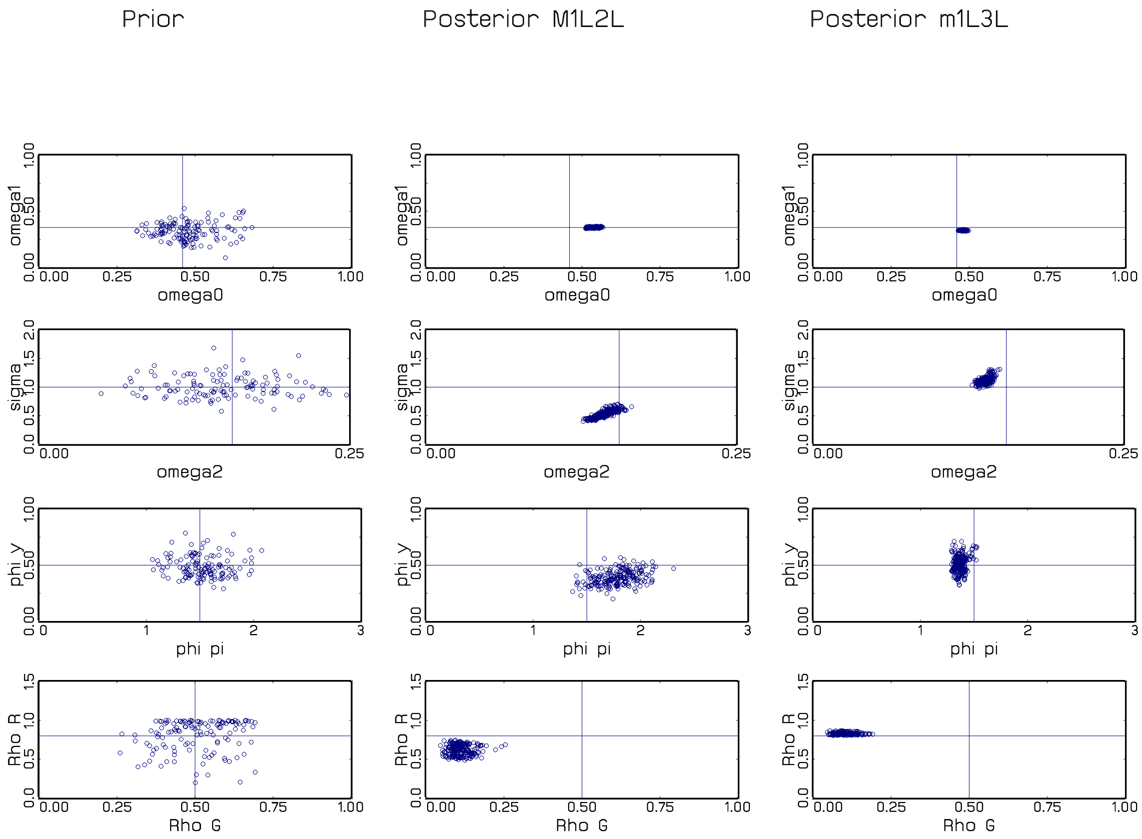
EMPIRICAL ISSUES (contd.)

Prior and Posterior Distributions

				Prior		M1L2L		M1L3L
Name	Range	Dist.	Mean	90%Itv	Post.	90% Itv	Post.	90% Itv
σ	RR^+	Gamma	1.00	[0.68, 1.32]	0.53	[0.42, 0.64]	1.12	[1.00, 1.21]
ϕ_π	RR^+	Gamma	1.52	[1.21, 1.85]	1.79	[1.51, 2.08]	1.37	[1.29, 1.43]
ϕ_y	RR^+	Gamma	0.49	[0.33, 0.65]	0.39	[0.28, 0.51]	0.52	[0.38, 0.64]
ρ_R	$[0, 1)$	Beta	0.80	[0.49, 1.00]	0.62	[0.51, 0.71]	0.83	[0.81, 0.86]
ρ_G	$[0, 1)$	Beta	0.50	[0.34, 0.67]	0.11	[0.06, 0.17]	0.10	[0.05, 0.14]
ρ_Z	$[0, 1)$	Beta	0.50	[0.33, 0.66]	0.39	[0.23, 0.53]	0.29	[0.17, 0.40]
$\omega 0$	$[0, 1)$	Beta	0.48	[0.35, 0.62]	0.54	[0.52, 0.56]	0.48	[0.47, 0.49]
$\omega 1$	$[0, 1)$	Beta	0.32	[0.19, 0.44]	0.36	[0.35, 0.36]	0.33	[0.33, 0.33]
$\omega 2$	$[0, 1)$	Beta	0.15	[0.07, 0.23]	0.14	[0.13, 0.16]	0.14	[0.13, 0.15]
σ_R	RR^+	InvGamma	1.77	[0.40, 3.13]	1.47	[1.07, 1.86]	0.87	[0.77, 0.97]
σ_G	RR^+	InvGamma	1.77	[0.41, 3.11]	0.16	[0.14, 0.18]	0.15	[0.13, 0.17]]
σ_Z	RR^+	InvGamma	1.78	[0.40, 3.12]	0.17	[0.15, 0.19]	0.20	[0.17, 0.22]

EMPIRICAL ISSUES (contd.)

Comparison of M1L2L and M1L3L



The solid lines denote the calibrated parameters used to simulate the data set.

CONCLUDING REMARKS

- The SDPC framework allows to link micro evidence on price setting behavior and macro evidence on inflation persistence.
- The SDPC framework allows to illustrate how inflation persistence varies in response to policy/structural shifts.
- The hybrid NKPC captures the macroeconomic dynamics fairly well as long as there is little or no state-dependent price-setting.
- Where do we stand? Too early to assess the worth of the SDPC for practical purposes. Challenges:
  - Calibration of SDPC: look at micro data to learn about price adjustment behavior.
  - Estimation of SDPC: tricky issue, no results yet.

CHARACTERIZATION OF THE SDPC (contd.)

Reduced form SDPC coefficients, $\pi_{ss} = 3\%$

$\mu'_j, \delta'_j, \psi'_j$ γ_j, η_j